# GLASS STRENGTH UNDER POINT LOADING

## Parametric Determination of Probability of Breakage for Typical Panel Dimensions

| | |
|---|---|
|  | **Joshua Schultz, Ph.D, P.E., LEED AP, ENV SP**<br>**Gonzaga University**<br>**schultzj@gonzaga.edu** |
|  | **John Knowles, PE SE**<br>**Stutzki Engineering**<br>**john.knowles@stutzkiengineering.com** |
|  | **Kedar Malusare**<br>**Stutzki Engineering**<br>**kedar.malusare@stutzkiengineering.com** |

### ABSTRACT

Structural glass is used in a range of building applications, and while the ASTM E1300 has recently been updated to provide a design method to account for probabilistic failure models for a range of geometries under uniform loads, there does not exist detailed design information for point loads. While, ASTM E2751 provides an allowable stress based on a theoretical probability of breakage (POB) of 8/1000 from the appendix of ASTM E1300, these allowable stresses have only a loose correlation to POB and are not accurate for all glass configurations. To address this, the authors explore the effect of point loads on glass strength using the glass failure prediction model (GFPM) from E1300 to determines the allowable stress (or allowable load) for common glass lite geometry based on a POB of 8/1000 and 1/1000. Results allow practitioners to benchmark designs for monolithic and PVB laminated glass based on various point-support boundary conditions.

The parametric study investigates the maximum principle stresses and stress concentrations of specimen types subjected to out-of-plane bending due to a point load over 26 cm$^2$ (4 in$^2$).The maximum point load is found for various 4-side supported glass lites at a POB of 8/1000 and 1/1000.

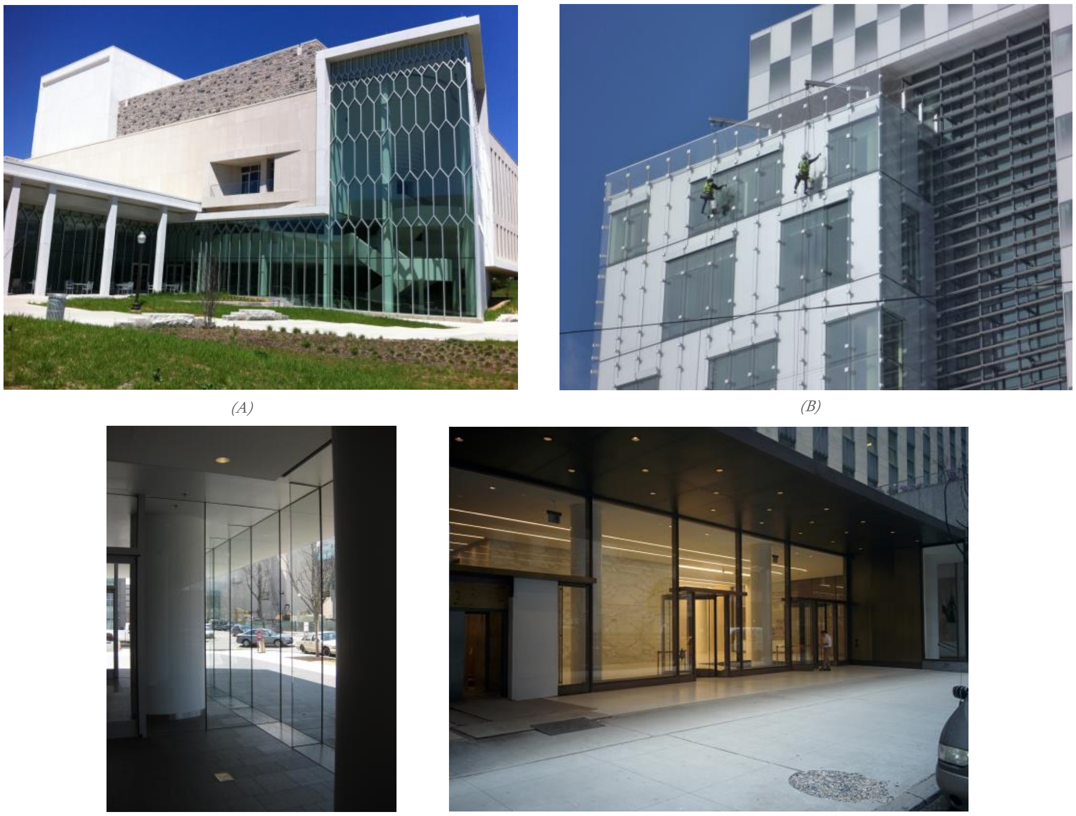
### KEYWORDS

Structural glass, glass engineering, component performance, structural design, experiment, point load

### INTRODUCTION

Structural glass applications, particularly façade and canopy elements, continue to grow in ubiquity as transparency and daylighting remain significant motivators of architectural design. However, current design procedures (i.e., elasticity and plate theories) and US design standards (e.g. ASTM E1300, 2016) are not directly applicable to structural glass with concentrated point loading. While ASTM E2751 (2017) does address point loads as design considerations it lacks a clear approach to determining strength, instead referencing ASTM E1300. The lack of clear design criteria for glass applications with point loading has resulted in designers relying on stress distributions and single largest maximum principle tensile stress (SLMPTS) as a guide for acceptable behavior which is often either unconservative or over-conservative – even when using low ultimate stress values to dictate design thicknesses. Development of the most recent ASTM E1300 has changed the approach to a range of structural glass applications due to the probabilistic-based failure prediction method explicitly presented in Annex A2, coupled with the acceptance of numerical modeling to obtain stresses and stress distributions. While the current ASTM E1300-16 still only contains non-factored load (NFL) charts for uniformly distributed loads, it is possible to establish a set of plots similar to those found in Annex A1 using the methodology presented in Annex A2. In an effort to provide meaningful design criteria to engineers and architects alike, a statistical glass failure prediction model (GFPM) is applied finite element analysis (FEA) data to provide design strength results for annealed and heat-strengthened (HS) specimens under point loading that provide a probability of breakage (POB) of 1 in 1,000 and 8 in 1,000, respectively. GFPM results are intended to 1) allow practitioners to benchmark designs for Annealed and HS monolithic glass based on various geometric conditions 2) provide strength of annealed and HS specimens for POB of 1/1000 and 8/1000 and 3) serve as a baseline methodology for more comprehensive, on-going efforts to create complete tables for designers. Additionally, a detailed description of the GFPM is provided with background information in order to assist designers with understanding of the necessity of a rigorous POB-based design approach compared to sole reliance on the SLMPTS.

The growth of glass as a façade component includes applications of skylights, glass floors and curtain walls. The industry trend is to produce larger glass lites to increase transparency and by extension the natural lite and sight lines inside the building.  As the need for larger glass is desired, loads from human impact have greater consequences than in the traditional applications of smaller glass panels. Impact loads can come from accidental impact from nearby pedestrians or impact while cleaning. For example, the Virginia Tech curtain wall (Fig. 1A), University of Baltimore curtain wall (Fig. 1B) which is subject to regular cleaning, University of Chicago Biomedical School glass wall with floor to ceiling glass (Fig. 1C) and the large line-supported glass panels of the American Girl Doll Store (Fig. 1D) each have considerable design differences, but all required consideration of an applied point load due to the large areas of glass which increase probability of breakage (POB).



(A)

(B)

(C)

(D)

*Figure 1: Projects are increasingly using large panels of glass which neccesitate evaluation of point loads (A) Virginia Tech curtain wall, (B) University of Baltimore curtain wall (C) University of Chicago Biomedical School glass wall and (D) large line-supported glass panels of the American Girl Doll Store. Photo courtesy Stutzki Engineering.*

A more obvious example would be the skylight glass used on the roof of the Aspen Art Museum which also is designed for pedestrian traffic (Fig. 2). This skylight needs to consider the concentrated point loads specified in ASTM E2751 (Standard

Practice for Design and Performance of Supported Glass Walkways) and references ASTM E1300 for calculation guidance, but the design charts in ASTM E1300 (Standard Practice for Determining Load Resistance of Glass in Buildings) are for uniform loads only.

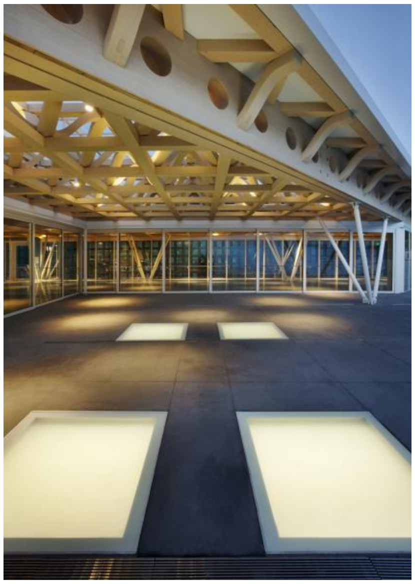


*Figure 2: The Aspen Art Museum, Aspen, Colorado is a typical application of line–supported, rectangular glass lites subjected to point loading in addition to uniformly distributed loads. Photo courtesy Stutzki Engineering.*

In order to address this design gap, the GFPM is applied to two common glass types (annealed and HS glass) and a range of aspect ratios using the same considerations as the provisions in the commonly referenced ASTM E1300 standard, but focuses on the unique results for point-loading as described in ASTM E2751. This method incorporates the relevant parameters (stress concentration due to Griffith surface flaws, load-duration effects, humidity, residual compressive surface stress, etc.,) on the principle stresses in order to provide designers with design strengths for specific POB for glass panels with point loads.

## *LITERATURE REVIEW ON PROBABILISTIC GLASS FAILURE*

In the United States, most structural glass is currently designed either directly or indirectly according to ASTM E1300 (2016). The nonfactored load (NFL) design charts in Annex A1 of E1300 allow for quick selection of minimum thickness for given panel geometry and desired load resistance of annealed glass. Design thicknesses for other glass types (heat strengthened (HS), fully tempered (FT), laminated glass (LG), insulating glass (IG), wired, and etched) are readily achieved by application of modification factors to the annealed selection. While this procedure appears straightforward, it is essential that designers understand the assumptions and limitations of the method. For example, the design charts in Annex A1 of ASTM E1300 are specifically for determination of the minimum thickness of window glass and are explicitly limited to rectangular geometries with a uniform load that are simply supported along at least one edge. Additionally, Appendix X6 offers “conservative” allowable stresses for use with finite element analysis for atypical glass geometries. However, the source of these “conservative” values is unknown, and there is only a loose correlation between these allowable stresses and actual POB of the glass lite. Since this limitation extends only to the charts in Annex A1 and not the procedure of Annex A2 it is important for designers to understand both the theory behind the design charts and methodology presented in ASTM E1300. The glass failure prediction model (GFPM) that serves as the basis for the NFL charts is used in this research to develop design strengths for POB of 1 in 1,000 and 8 in 1,000 for annealed and HS rectangular lites exposed to point loads. The efficient design of a brittle material with statistical failure behavior (i.e., glass) requires an accurate determination of a multitude of factors: SLMPTS, stress distributions, surface flaw characterization, residual compressive surface stress (RCSS), load-duration, temperature and humidity effects, etc (Schultz et al., 2017). In order to accurately use the ASTM E1300 design method it is important to understand each term and how it has been integrated into the procedure over time.

The brittle and probabilistic nature of glass as a material means that it often fails under stresses significantly lower than the ultimate stress. As early as 1920, it was hypothesized that this was a result of microscopic surface flaws that created stress concentrations which promoted crack growth (Griffith, 1920). Evidence of this behavior was readily observed in failures of all glass types, as the initiation of the failure was often not coincident with the SLMPTS (Natividad et al., 2015). The distribution and orientation of these surface flaws was modelled using the statistical multiple parameter fitting method presented by Weibull (1939). In addition to surface flaws, subsequent research attributed the apparent reduction in annealed glass strength to load duration and presence of humidity (Charles 1958). Many experimental initiatives tested annealed and HS glass specimens to failure in an effort to quantify this behavior (Orr, 1957; Bowles and Sugarman, 1962). The results of these tests combined with successful empirical designs formed the basis for a series of design guidelines developed by the glass industry for window glass in the early to mid-1960s.

A significant change in glass design occurred when Beason and Morgan (1984) combined the effects of surface flaws (orientation and distribution) and humidity into a single glass failure prediction method (GFPM) using a two parameter Weibull distribution and determination of SLMPTS based on the work of Vallabhan et al., (1985, 1987). This research reframed glass strength fully in terms of probability of breakage (POB) rather than ultimate strengths (which were readily mistaken for a type of allowable stress design). The method developed by Beason and Morgan assumes that the glass strength is 1) governed by tensile stress failure and 2) independent of load duration (not necessarily an accurate assumption for all types of glass, i.e., laminated glass). This methodology served (and continues to serve) as the fundamental design approach for glass design procedures and charts in every ASTM E1300 standard from 1989 to 2016. However, subsequent research by Morse and Norville (2011, 2012) has relaxed assumptions of glass type (annealed, HS and FT) and geometry used in the ASTM E1300 charts by creating more general forms of the GFPM equation. In fact, Morse and Norville (2012) presented the explicit form of the risk function, B, not found in ASTM E1300, used by Walker and Muir (1984) for consideration of edge flaws in annealed glass fins. More recently, Afolabi, Norville and Morse (2016) provided a detailed experimental study of HS line supported glass with discussion of RCSS and equivalent failure load (EFL). However, none of these studies focused on point-loads. There has been some research into the aapplication of the GFPM to non-traditional applications like point-supported glass (PSG) was addressed by Cervenka et al., (2016). That research focused on experimental stresses and stress distribution near holes in monolithic and laminated FT glass to determine influence of stress concentrations. The work was extended by Schultz et al., (2017) to create 3-second failure load curves and determination of best-fit m- and k-values for PSG.

## METHOD

A series of parametric finite element models were run using Abaqus FEA software (Abaqus, 2017) to obtain the maximum principle stresses and stress distributions. The modelling approach followed the methods outlined by Schultz et al. (2012) and Cervenka et al., (2016), both of which were benchmarked to experimental data to determine accuracy of numerical results. The models were line-supported, rectangular panels modeled with quarter panel symmetry. Out-of-plane displacement of the glass edges were fixed and the other BCs were addressed through the symmetry conditions. Due to the importance of boundary conditions (Schultz et al., 2012), contact interactions were evaluated BC's with multiple materials and found to have a negligible impact on the stress results; consequently, the models were created with BC's applied directly to the edge of the glass. Both annealed and HS panels were modeled as industry-standard linear elastic material with Young's modulus, E, of 72 GPa and Poisson's ratio, v, as 0.22 for a range of aspect rations (Fig. 3).

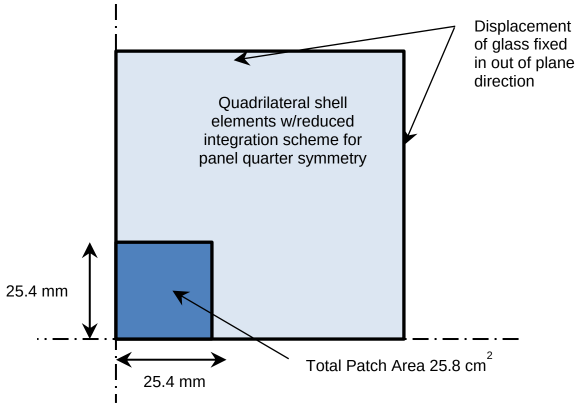


*Figure 3: Modeling geometry and options are shown for the parametric study*

A geometrically non-linear analysis was used to account for displacements that are large with respect to the thickness. A quadrilateral shell element with reduced integration scheme, hourglass control, finite membrane strains (Abaqus: S4R) was used as recommended by industry best-practices and previous research (Cervenka et al., 2016, Schultz et al., 2012). Point loads were applied via a 1”x 1” sub-meshed area that was subjected to uniform pressure load.

Once the SMPLTS and stress distributions were obtained from the FEA, the GFPM described by Morse and Norville (2011) was used to determine the POB for annealed and HS glass lites for POB of 8/1000 and 1/1000, respectively. The probability of breakage of a glass specimen using the GFPM is given as:

$$P_b = 1 - e^{-B} \tag{1}$$

where, B for a 3-sec EFL is defined in numerical form as:

$$B = k \cdot \sum_{i=1}^{N} \left( \left( c_i \cdot \left( \frac{t_d}{3s} \right)^{\frac{1}{n}} \cdot \sigma_{max} - RCSS \right)^{m} \cdot L_i \right) \tag{2}$$

with N as the numbers of nodes in the numerical model, $c_i$ as the biaxial correction factor (in terms of $r_i$, $\sigma_{max}$, $\sigma_{min}$), $\sigma_{max}$ representing the maximum principal stress, $L_i$ is the tributary length of the element, n = 16 and m and k are surface flaw parameters are based on a 60s duration (per ASTM E1300-16 A2).

## DATA

Maximum, deflections, U, principle stresses, S1, and stress distributions were found for the rectangular panels of varying aspect ratios (Figs 4-6). The stress distributions and deflected shapes of the panels follow symmetric patterns that are expected both from the symmetric BC’s as well as from classical plate theory. Representative values of stress and deflection are shown for the loads at failure.

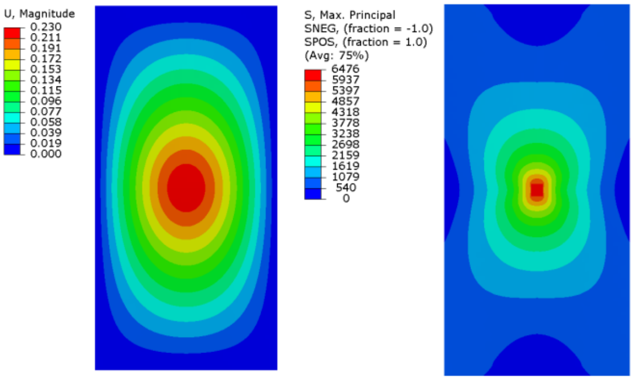


*Figure 4: Representative (a) deflections and (b) maximum principle stresses are shown for 75 cm x 150 cm panels*

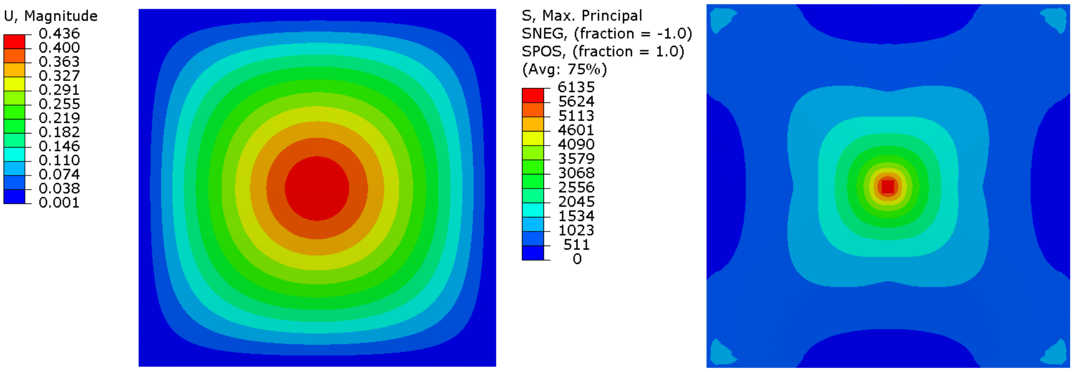


*Figure 5: Representative (a) deflections and (b) maximum principle stresses are shown for 150 cm x 150 cm panels*

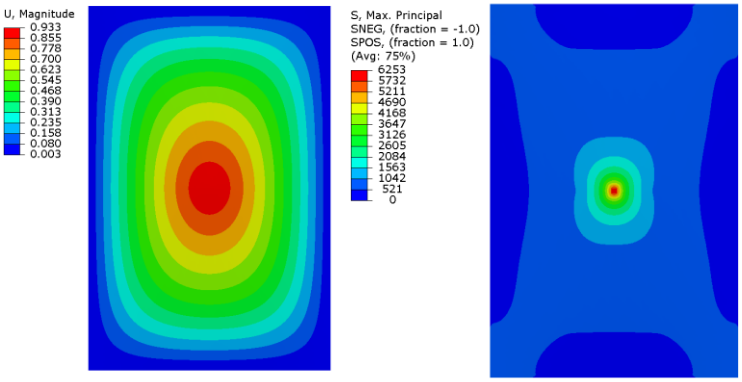


*Figure 6: Representative (a) deflections and (b) maximum principle stresses are shown for 245 cm x 370 cm panels*

The stress results were used in combination with the GFPM described by Eqns (1) and (2) to obtain design stresses for POB of 8/1000 (Fig. 7) and 1/1000 (Fig. 8) for annealed glass and HS glass (Fig. 9-10) for a 3-second load duration.

| Aspect Ratio | Dimensions (mm) | | Load (kN) | Stress (kPa) | POB | Def (mm) |
|---|---|---|---|---|---|---|
| | X | Y | | | | |
| 1 | 254 | 254 | 0.83 | 42.94 | 0.008 | 23.4 |
| 1 | 180 | 180 | 0.81 | 42.57 | 0.008 | 16.4 |
| 1 | 127 | 127 | 0.78 | 42.30 | 0.008 | 11.1 |
| 1 | 90 | 90 | 0.76 | 42.35 | 0.008 | 7.1 |
| 1.5 | 207 | 311 | 0.81 | 43.11 | 0.008 | 23.7 |
| 1.5 | 147 | 220 | 0.78 | 43.04 | 0.008 | 16.3 |
| 1.5 | 104 | 156 | 0.75 | 43.17 | 0.008 | 10.7 |
| 1.5 | 73 | 110 | 0.75 | 43.69 | 0.008 | 6.7 |
| 2 | 180 | 359 | 0.77 | 43.68 | 0.008 | 23.6 |
| 2 | 127 | 254 | 0.74 | 43.68 | 0.008 | 15.5 |
| 2 | 90 | 180 | 0.72 | 43.63 | 0.008 | 9.6 |
| 2 | 64 | 127 | 0.75 | 44.65 | 0.008 | 5.8 |

*Figure 7: Load and stress results at POB of 8/1000 shown for 6 mm thick annealed glass lites*

| Aspect Ratio | Dimensions (mm) | | Load (kN) | Stress (kPa) | POB | Def (mm) |
|---|---|---|---|---|---|---|
| | X | Y | | | | |
| 1 | 254 | 254 | 0.56 | 31.16 | 0.001 | 19.3 |
| 1 | 180 | 180 | 0.54 | 30.79 | 0.001 | 13.3 |
| 1 | 127 | 127 | 0.52 | 30.60 | 0.001 | 8.7 |
| 1 | 90 | 90 | 0.53 | 30.73 | 0.001 | 5.5 |
| 1.5 | 207 | 311 | 0.54 | 31.26 | 0.001 | 19.4 |
| 1.5 | 147 | 220 | 0.52 | 31.16 | 0.001 | 13.0 |
| 1.5 | 104 | 156 | 0.51 | 31.34 | 0.001 | 8.3 |
| 1.5 | 73 | 110 | 0.52 | 31.77 | 0.001 | 5.1 |
| 2 | 180 | 359 | 0.52 | 31.61 | 0.001 | 18.8 |
| 2 | 127 | 254 | 0.50 | 31.74 | 0.001 | 12.1 |
| 2 | 90 | 180 | 0.50 | 31.98 | 0.001 | 7.4 |
| 2 | 64 | 127 | 0.53 | 32.52 | 0.001 | 4.3 |

*Figure 8: Load and stress results at POB of 1/1000 shown for 6 mm thick annealed glass lites*

| Aspect Ratio | Dimensions (mm) | | Load (kN) | Stress (kPa) | POB | Def (mm) |
|---|---|---|---|---|---|---|
| | X | Y | | | | |
| 1 | 254 | 254 | 1.61 | 72.46 | 0.008 | 31.9 |
| 1 | 180 | 180 | 1.59 | 72.35 | 0.008 | 22.8 |
| 1 | 127 | 127 | 1.54 | 72.17 | 0.008 | 16.0 |
| 1 | 90 | 90 | 1.48 | 71.91 | 0.008 | 10.7 |
| 1.5 | 207 | 311 | 1.57 | 73.07 | 0.008 | 32.7 |
| 1.5 | 147 | 220 | 1.54 | 73.15 | 0.008 | 23.1 |
| 1.5 | 104 | 156 | 1.48 | 73.26 | 0.008 | 15.8 |
| 1.5 | 73 | 110 | 1.42 | 73.42 | 0.008 | 10.3 |
| 2 | 180 | 359 | 1.52 | 74.06 | 0.008 | 33.4 |
| 2 | 127 | 254 | 1.47 | 74.44 | 0.008 | 22.9 |
| 2 | 90 | 180 | 1.40 | 74.42 | 0.008 | 14.9 |
| 2 | 64 | 127 | 1.37 | 74.81 | 0.008 | 9.2 |

*Figure 9: Load and stress results at POB of 8/000 shown for 6 mm thick HS glass lites*

| Aspect Ratio | Dimensions (mm) | | Load (kN) | Stress (kPa) | POB | Def (mm) |
|---|---|---|---|---|---|---|
| | X | Y | | | | |
| 1 | 254 | 254 | 1.28 | 60.52 | 0.001 | 28.7 |
| 1 | 180 | 180 | 1.26 | 60.27 | 0.001 | 20.4 |
| 1 | 127 | 127 | 1.22 | 60.16 | 0.001 | 14.1 |
| 1 | 90 | 90 | 1.17 | 59.89 | 0.001 | 9.4 |
| 1.5 | 207 | 311 | 1.25 | 60.85 | 0.001 | 29.3 |
| 1.5 | 147 | 220 | 1.22 | 61.00 | 0.001 | 20.5 |
| 1.5 | 104 | 156 | 1.17 | 61.07 | 0.001 | 13.9 |
| 1.5 | 73 | 110 | 1.13 | 61.35 | 0.001 | 8.9 |
| 2 | 180 | 359 | 1.21 | 61.93 | 0.001 | 29.8 |
| 2 | 127 | 254 | 1.16 | 61.91 | 0.001 | 20.1 |
| 2 | 90 | 180 | 1.12 | 62.21 | 0.001 | 12.9 |
| 2 | 64 | 127 | 1.10 | 62.48 | 0.001 | 7.9 |

*Figure 10: Load and stress results at POB of 1/1000 shown for 6 mm thick HS glass lites*

## EXPLANATION

The design loads and related stresses presented in Figs 7-10, provide designers a simple reference for selection of geometries to meet desired POB for annealed and HS lites of 5.56 mm according to the GFPM used in accordance with the procedure in ASTM E1300 Annex 2 for point loads. For the four side supported 5.56 mm thick specimens, there was relatively weak dependence of loads/stresses on panel dimensions with a maximum observed change of about 5% in breaking load from the aspect ratio of 1:1 to 1:2.

The design values presented far exceed commonly used (i.e., standard) industry values but still fall within the limits of 8/1000 and 1/1000 POB, respectively. The FEA analysis shows significant membrane action due to the four side supports, and it is expected that results may be significantly lower for three and two side supports. Comparing the results to E1300 appendix values shows that ASTM E1300 has conservative design loads.

## CONCLUSION AND FUTURE WORK

The results presented are for four side supported, 6 mm thick, annealed and HS glass types. Ongoing studies are being completed to build out a series of design tables for point loads for a range of common thicknesses (6 – 12 mm), glass types (annealed, HS and FT) as well as support types (2, 3, and 4 side support).

## ACKNOWLEDGMENTS

The authors gratefully acknowledge Stutzki Engineering for the use of images.

## REFERENCES

ABAQUS: Multi-Purpose Finite Element Analysis Software, Dassault Systems. 2017

Afolabi, Bolaji, H. Scott Norville, and Stephen M. Morse. "Experimental study of weathered tempered glass plates from the Northeastern United States." *Journal of Architectural Engineering* 22, no. 3 (2016): 04016010.

ASTM International. ASTM E1300-16 *Standard practice for determining load resistance of glass in buildings.* West Conshohocken, Pennsylvania, USA. ASTM; 2016. www.astm.org p. 1-62.

ASTM International. ASTM E2751-17a. *Standard Practice for Design and Performance of Supported Glass Walkways.* West Conshohocken, Pennsylvania, USA. ASTM; 2017. www.astm.org p. 1-8.

Beason, W. Lynn, and James R. Morgan. "Glass failure prediction model." *Journal of Structural Engineering* 110, no. 2 (1984): 197-212.

Bowles, R., and B. Sugarman. "The strength and deflection characteristics of large rectangular glass panels under uniform pressure." *Glass Technology* 3, no. 5 (1962): 156-170.

Cervenka, J., Schultz, J.A., Stahl, D., and Knowles, J. “Strength of point-supported glass.” In *Proceedings of the 1st Façade Tectonics World Congress*; *Tectonics Press Publications.,* pp. 381-389. 2016.

Charles, R. J. "Static fatigue of glass. I and II." *Journal of Applied Physics* 29, no. 11 (1958): 1549-1553.

Griffith, Alan A. "The phenomena of rupture and flow in solids." *Philosophical transactions of the royal society of london. Series A, containing papers of a mathematical or physical character* 221 (1921): 163-198.

Morse, S. and Norville, H. “An analytical method for determining window glass strength.” In *Proceedings of Glass Performance Days 2011*; *GPD Press 9*; pp. 480-482. 2011.

Morse, Stephen M., and H. Scott Norville. "Design methodology for determining the load resistance of heat-treated window glass." *Journal of Architectural Engineering* 18, no. 1 (2011): 42-51.

Natividad, Kayla, Stephen M. Morse, and H. Scott Norville. "Fracture Origins and Maximum Principal Stresses in Rectangular Glass Lites." *Journal of Architectural Engineering* 22, no. 2 (2015): 04015014.

Orr, Leighton. "Engineering properties of glass." *Publication* 478 (1957): 51-61. Bowles and Sugarman, 1962;

Schultz, J.A., Knowles, J., and Morse, S., “Glass Failure Prediction Model for Out-of-Plane Bending of Waterjet-Drilled Holes,” In *Proceedings of the 39th IABSE Symposium, International Associated of Bridge and Structural Engineers*, Vancouver, Canada. 2017.

Schultz, Joshua, Douglas Stahl, and Christian Stutzki. "Experimental investigation of numerical design method for point-supported glass." *Journal of Architectural Engineering* 18, no. 3 (2012): 223-232.

Vallabhan, CV Girija, Bob Yao-Ting Wang, Gee David Chou, and Joseph E. Minor. "Thin glass plates on elastic supports." *Journal of Structural Engineering* 111, no. 11 (1985): 2416-2426.

Vallabhan, CV Girija, Joseph E. Minor, and Sesha R. Nagalla. "Stresses in layered glass units and monolithic glass plates." *Journal of structural Engineering* 113, no. 1 (1987): 36-43.

Walker, G. R., and L. H. Muir. "Investigation of the Bending Strength of Glass Louvre Blades." In *Ninth Australasian Conference on the Mechanics of Structures and Materials, ACMSM 9.*, pp. 122-126. 1984.

Weibull, Waloddi. "A statistical theory of the strength of materials." *Ingeniors Vetenskaps Akademien* (1939).