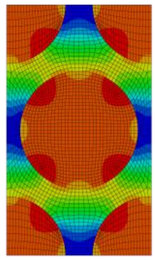
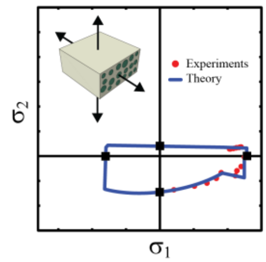
ENERGY CONSERVING VOLUME AVERAGE STRESSES TO PREDICT COMPOSITE FAILURE



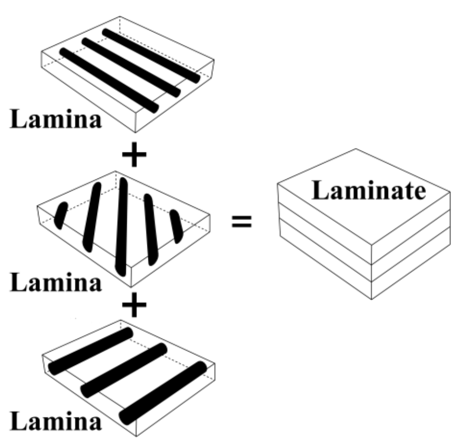


Kedar A. Malusare,
Masters Thesis Defense
4/13/2014
Department of Mechanical Engineering, University of Wyoming.

UNIVERSITY OF WYOMING

What are composites ?

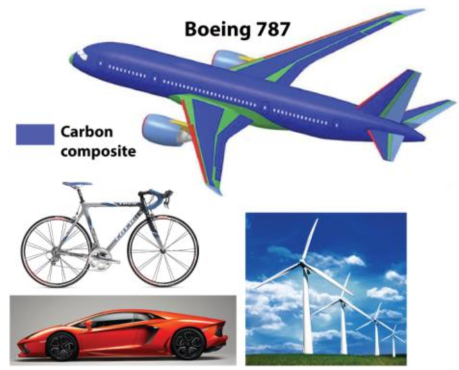
- Heterogeneous mixtures of two or more homogeneous phases
- This work focuses on unidirectional fiber reinforced polymers
- Fibers are embedded in a polymer matrix to obtain a lamina
- Fibers – strength
  Matrix – provides stability & transmits load among fibers
- Laminae are stacked together to form laminates



UNIVERSITY OF WYOMING

Where are they used ?

- Composites are widely used in various industries
- Popular in aerospace, wind energy sector, automobile & recreation etc.
- Have high specific stiffness, high specific strengths etc.
- Tailor their material properties according to needs of end product
- Increase strength, reduce weight and costs

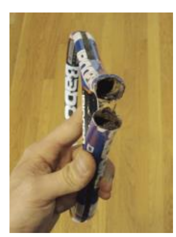


Do composite materials fail ?

Failure of composites

Blade[1]

Rudder[2]



Tennis racket[3]

Pressure vessel[4]

Bike frame[5]

1.http://www.eastcountymagazine.org/node/2734
2.http://www.yachtsurvey.com/composite_troubles_in_aircraft.htm
3.http://blog.tennishub.com/blog
4.http://www.immt.pwr.wroc.pl/~gasior/Researches/Laboratory/laboratory.htm
5.http://www.bustedcarbon.com/2010_06_01_archive.html

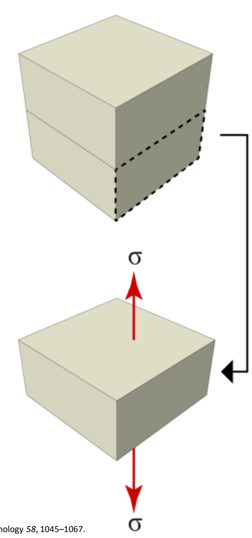
Overview

PROBLEM / SOLUTION

1. Failure modeling techniques - ***HOW to predict failure ?***
2. Missing strain energy - "Interaction energy"
3. Investigate the nature of "Interaction energy" using FEA
4. Failure modeling using volume average constituent stresses
5. Modeling fluctuations and augmenting volume average quantities
6. Failure modeling using energy conserving constituent stresses
7. Summary/Conclusions
8. Future work

Mesomodeling

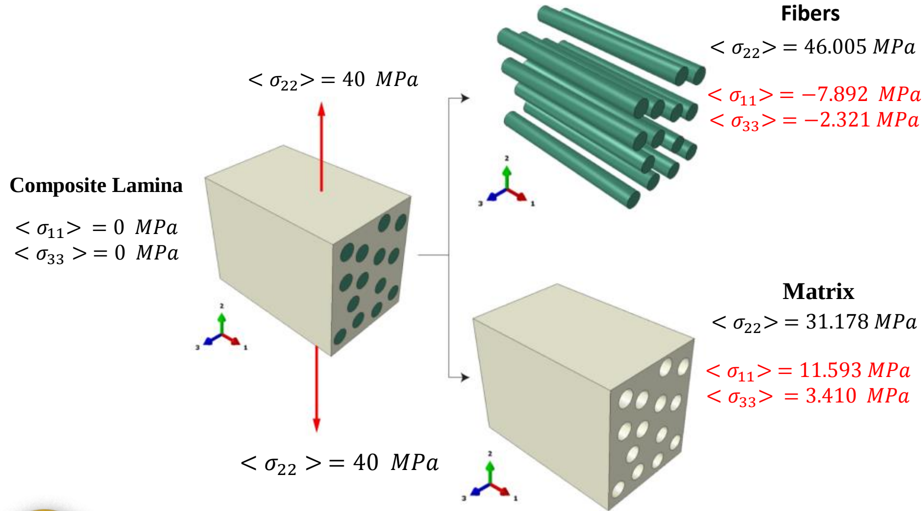
- Considers lamina layers (plies) as building blocks of laminates
- Use volume average lamina quantities (stresses & strains) to predict failure
- Examples Maximum stress/strain, Tsai-Wu[6], Hashin[7], Christensen[8], Puck[9] etc.
- Failure prediction remains inadequate



Do <u>lamina</u> quantities capture the true stress/strain state in a constituent ?

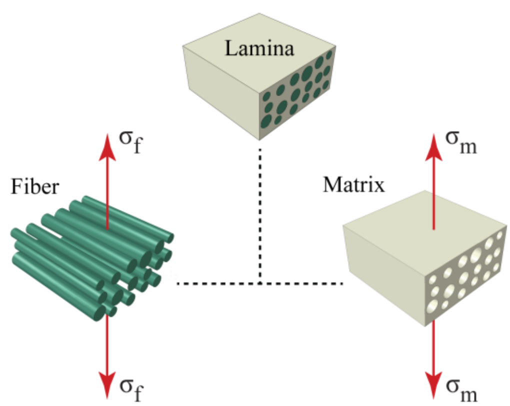
6. Tsai, S.W., and Wu, E.M. (1971). A general theory of strength for anisotropic materials. Journal of Composite Materials *5*, 58–80.
7.Hashin, Z., and Rotem, A. (1973). A fatigue failure criterion for fiber reinforced materials. Journal of Composite Materials *7*, 448–464.
8. Christensen, R. (1997). Stress based yield/failure criteria for fiber composites. International Journal of Solids and Structures *34*, 529–543.
9. Puck, A., and Schürmann, H. (1998). Failure analysis of FRP laminates by means of physically based phenomenological models. Composites Science and Technology *58*, 1045–1067.

Volume average constituent stresses



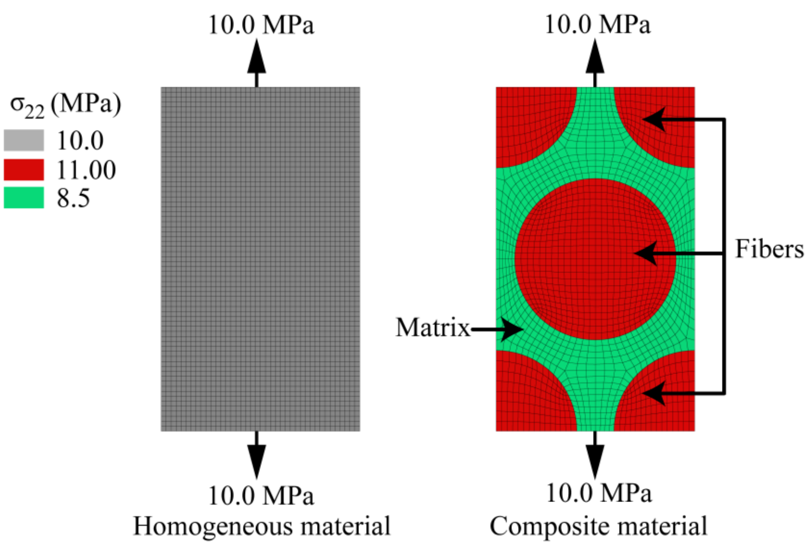
Multiscale micromechanical modeling

- Use average constituent quantities to predict failure
- Can apply constituent level physics
- Can predict the response of the entire composite using just constituent properties
- Examples are Chamis[10], Mayes[11], Huang[12] & Tsai-Ha[13].

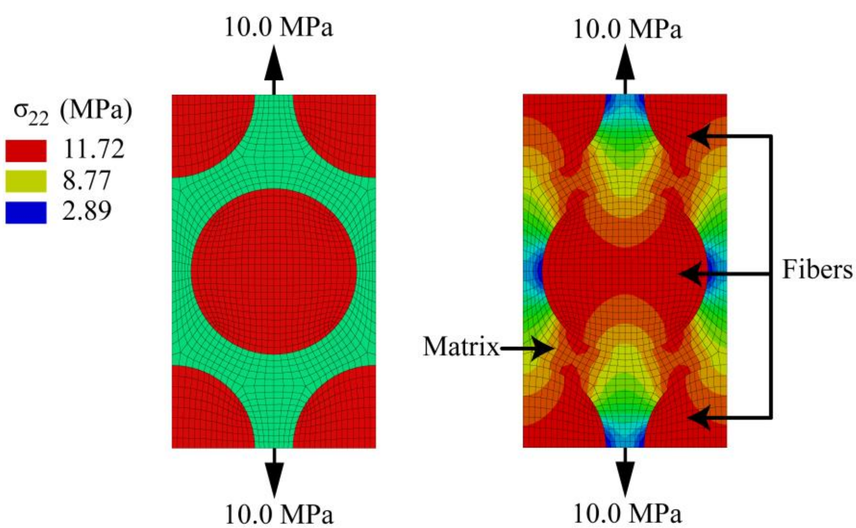


10. Gotsis, P., Chamis, C.C., and Minnetyan, L. (1998). Prediction of composite laminate fracture: micromechanics and progressive fracture. Composites Science and Technology *58*, 1137–1149.
11. Mayes, J.S., and Hansen, A.C. (2004). Composite laminate failure analysis using multicontinuum theory. Composites Science and Technology *64*, 379–394.
12. Huang, Z.-M. (2004). A bridging model prediction of the ultimate strength of composite laminates subjected to biaxial loads. Composites Science and Technology *64*, 395–448.
13. Huang, Y., Jin, C., and Ha, S.K. (2013). Strength prediction of triaxially loaded composites using a progressive damage model based on micromechanics of failure. Journal of Composite Materials *47*, 777–792.

Volume average constituent stresses



Distribution of stress in constituents



Overview

1. Failure modeling techniques - HOW to predict failure ?

PROBLEM

2. Missing strain energy - "Interaction energy"
3. Investigate the nature of "Interaction energy" using FEA
4. Failure modeling using volume average constituent stresses

SOLUTION

5. Modeling fluctuations and augmenting volume average quantities
6. Failure modeling using energy conserving constituent stresses
7. Summary/Conclusions
8. Future work

Strain energy comparison

$$U = \frac{1}{2} \sigma_{ij}{}^{c} \varepsilon_{ij}{}^{c} V_c$$

$$U_f = \frac{1}{2} \sigma_{ij}{}^{f} \varepsilon_{ij}{}^{f} V_f \qquad U_m = \frac{1}{2} \sigma_{ij}{}^{m} \varepsilon_{ij}{}^{m} V_m$$

$$U > U_f + U_m$$

$$U = \left(U_f + U_m\right) + \Delta U$$

where ΔU is the missing energy.

Interaction energy

$$U_f = \frac{1}{2}\,\sigma_{ij}{}^{f}\,\varepsilon_{ij}{}^{f}\,V_f + \Phi_f V_f$$

$$U_m = \frac{1}{2}\,\sigma_{ij}{}^{m}\,\varepsilon_{ij}{}^{m}\,V_m + \Phi_m V_m$$

$$\Delta U = \Phi_f V_f + \Phi_m V_m$$

where ΔU is the 'Interaction Energy'.

Interaction energy

$$\Phi_f = \frac{1}{2}\int_{V_f} \tilde{\boldsymbol{\sigma}}_{ij}^{f}\,\tilde{\varepsilon}_{ij}^{f}\,dV_f \qquad \Phi_m = \frac{1}{2}\int_{V_m} \tilde{\boldsymbol{\sigma}}_{ij}^{m}\,\tilde{\varepsilon}_{ij}^{m}\,dV_m$$

$$\Phi_f = \frac{1}{2}\int_{V_f} C_{ijkl}\,\tilde{\varepsilon}_{ij}^{f}\,\tilde{\varepsilon}_{kl}^{f}\,dV_f \qquad \Phi_m = \frac{1}{2}\int_{V_m} C_{ijkl}\,\tilde{\varepsilon}_{ij}^{m}\,\tilde{\varepsilon}_{kl}^{m}\,dV_m$$

Assuming transverse isotropy and expanding in *i*,*j*,*k* and, *l* yields

Expression for Interaction energy

$$\Phi_f = \frac{1}{2}\left[\begin{array}{l} C_{11}^{f}\left\langle \left(\tilde{\varepsilon}_{11}^{f}\right)^2\right\rangle + C_{22}^{f}\left\langle \left(\tilde{\varepsilon}_{22}^{f}\right)^2\right\rangle + C_{33}^{f}\left\langle \left(\tilde{\varepsilon}_{33}^{f}\right)^2\right\rangle + 2C_{12}^{f}\left\langle \tilde{\varepsilon}_{11}^{f}\cdot\tilde{\varepsilon}_{22}^{f}\right\rangle + 2C_{13}^{f}\left\langle \tilde{\varepsilon}_{11}^{f}\cdot\tilde{\varepsilon}_{33}^{f}\right\rangle + \\ 2C_{23}^{f}\left\langle \tilde{\varepsilon}_{22}^{f}\cdot\tilde{\varepsilon}_{33}^{f}\right\rangle + C_{12}^{f}\left\langle \left(\tilde{\gamma}_{12}^{f}\right)^2\right\rangle + C_{13}^{f}\left\langle \left(\tilde{\gamma}_{13}^{f}\right)^2\right\rangle + C_{23}^{f}\left\langle \left(\tilde{\gamma}_{23}^{f}\right)^2\right\rangle \end{array}\right]$$

$$\Phi_m = \frac{1}{2}\left[\begin{array}{l} C_{11}^{m}\left\langle \left(\tilde{\varepsilon}_{11}^{m}\right)^2\right\rangle + C_{22}^{m}\left\langle \left(\tilde{\varepsilon}_{22}^{m}\right)^2\right\rangle + C_{33}^{f}\left\langle \left(\tilde{\varepsilon}_{33}^{m}\right)^2\right\rangle + 2C_{12}^{m}\left\langle \tilde{\varepsilon}_{11}^{m}\cdot\tilde{\varepsilon}_{22}^{m}\right\rangle + 2C_{13}^{m}\left\langle \tilde{\varepsilon}_{11}^{m}\cdot\tilde{\varepsilon}_{33}^{m}\right\rangle + \\ 2C_{23}^{m}\left\langle \tilde{\varepsilon}_{22}^{m}\cdot\tilde{\varepsilon}_{33}^{m}\right\rangle + C_{12}^{m}\left\langle \left(\tilde{\gamma}_{12}^{m}\right)^2\right\rangle + C_{13}^{m}\left\langle \left(\tilde{\gamma}_{13}^{m}\right)^2\right\rangle + C_{23}^{m}\left\langle \left(\tilde{\gamma}_{23}^{m}\right)^2\right\rangle \end{array}\right]$$

Expression for Interaction energy

$$\Phi_f = \frac{1}{2}\left[\begin{array}{l} S_{11}^{f}\left\langle \left(\tilde{\sigma}_{11}^{f}\right)^2\right\rangle + S_{22}^{f}\left\langle \left(\tilde{\sigma}_{22}^{f}\right)^2\right\rangle + S_{33}^{f}\left\langle \left(\tilde{\sigma}_{33}^{f}\right)^2\right\rangle + 2S_{12}^{f}\left\langle \tilde{\sigma}_{11}^{f}\cdot\tilde{\varepsilon}_{22}^{f}\right\rangle + 2S_{13}^{f}\left\langle \tilde{\sigma}_{11}^{f}\cdot\tilde{\sigma}_{33}^{f}\right\rangle + \\ 2S_{23}^{f}\left\langle \tilde{\sigma}_{22}^{f}\cdot\tilde{\sigma}_{33}^{f}\right\rangle + S_{12}^{f}\left\langle \left(\tilde{\tau}_{12}^{f}\right)^2\right\rangle + S_{13}^{f}\left\langle \left(\tilde{\tau}_{13}^{f}\right)^2\right\rangle + S_{23}^{f}\left\langle \left(\tilde{\tau}_{23}^{f}\right)^2\right\rangle \end{array}\right]$$

$$\Phi_m = \frac{1}{2}\left[\begin{array}{l} S_{11}^{m}\left\langle \left(\tilde{\sigma}_{11}^{m}\right)^2\right\rangle + S_{22}^{m}\left\langle \left(\tilde{\sigma}_{22}^{m}\right)^2\right\rangle + S_{33}^{m}\left\langle \left(\tilde{\sigma}_{33}^{m}\right)^2\right\rangle + 2S_{12}^{m}\left\langle \tilde{\sigma}_{11}^{m}\cdot\tilde{\varepsilon}_{22}^{m}\right\rangle + 2S_{13}^{m}\left\langle \tilde{\sigma}_{11}^{m}\cdot\tilde{\sigma}_{33}^{m}\right\rangle + \\ 2S_{23}^{m}\left\langle \tilde{\sigma}_{22}^{m}\cdot\tilde{\sigma}_{33}^{m}\right\rangle + S_{12}^{m}\left\langle \left(\tilde{\tau}_{12}^{m}\right)^2\right\rangle + S_{13}^{m}\left\langle \left(\tilde{\tau}_{13}^{m}\right)^2\right\rangle + S_{23}^{m}\left\langle \left(\tilde{\tau}_{23}^{m}\right)^2\right\rangle \end{array}\right]$$

$$\Delta U = \Phi_f V_f + \Phi_m V_m$$

How does it depend on <u>fiber volume fraction</u> , <u>properties of the materials</u> or <u>applied load state</u>?

Overview

1. Failure modeling techniques - HOW to predict failure ?

PROBLEM

2. Missing strain energy - "Interaction energy"

3. Investigate the nature of "Interaction energy" using FEA

4. Failure modeling using volume average constituent stresses

SOLUTION

5. Modeling fluctuations and augmenting volume average quantities

6. Failure modeling using energy conserving constituent stresses

7. Summary/Conclusions

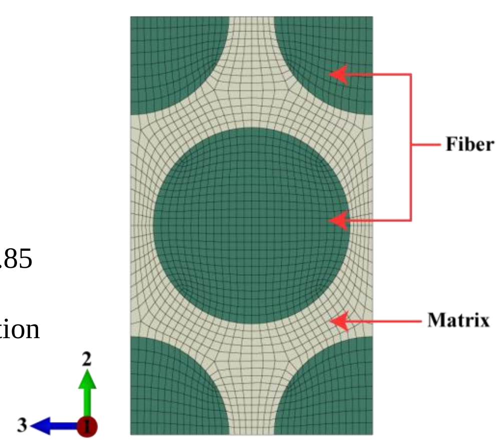
8. Future work

FEA model

- Representative Volume Element (RVE) with hexagonal fiber packing.
- Fiber material - **Carbon**

Three parametric studies:

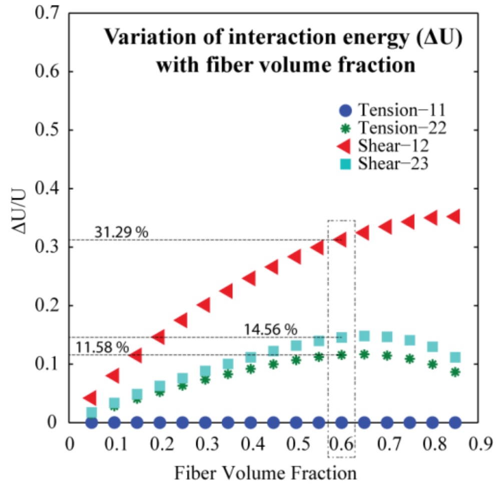
1. Fiber VF varied from 0.05 to 0.85
2. Matrix modulus varied as function of fiber modulus
3. Five types of biaxial loads



K. A. Malusare and R. S. Fertig, American Institute of Aeronautics and Astronautics, 2014.

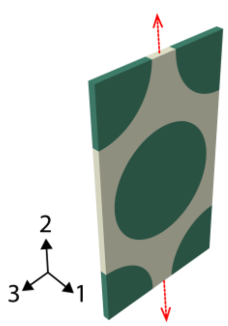
Effect of fiber volume fraction on interaction energy

- Matrix modulus 1% (2.35 GPa)
- Strongly dependent on the loading
- Maximum for shear-12 & negligible for tension-11
- For VF 0.6 ΔU is about 30% for shear-12.



K. A. Malusare and R. S. Fertig, American Institute of Aeronautics and Astronautics, 2014.

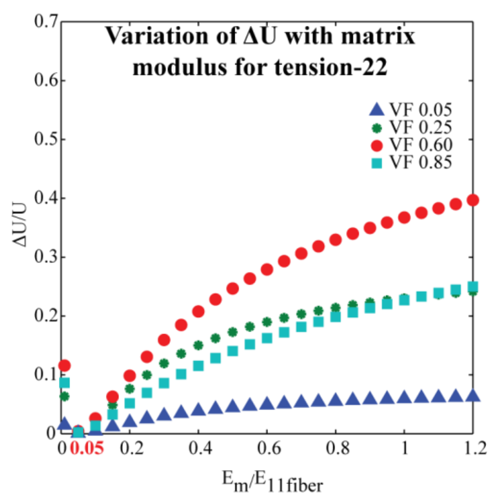
Effect of material properties on interaction energy



$E_{22_{matrix}} = 5\% \, E_{11_{fiber}}$

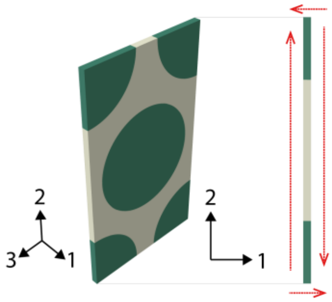
$E_{22_{matrix}} = 11.75\ GPa$
$E_{22_{fiber}} = 14.00\ GPa$

Interaction Energy is minimum



K. A. Malusare and R. S. Fertig, American Institute of Aeronautics and Astronautics, 2014.

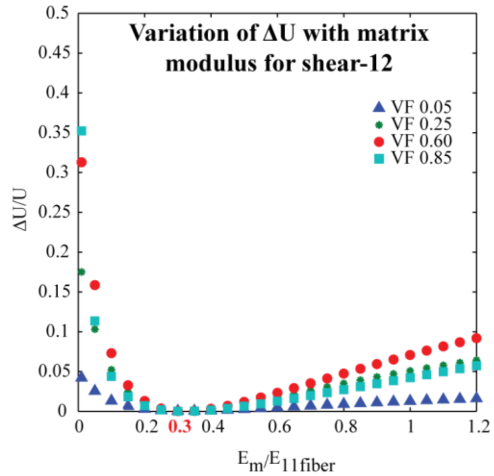
Effect of material properties on interaction energy



$E_{22_{matrix}} = 30\% \, E_{11_{fiber}} = 70.5 \, GPa$

$G_{12_{matrix}} = 26.31 \, GPa$
$G_{12_{fiber}} = 28.00 \, GPa$

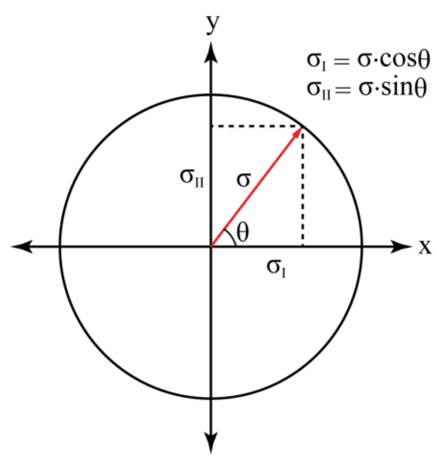
Interaction energy is minimum



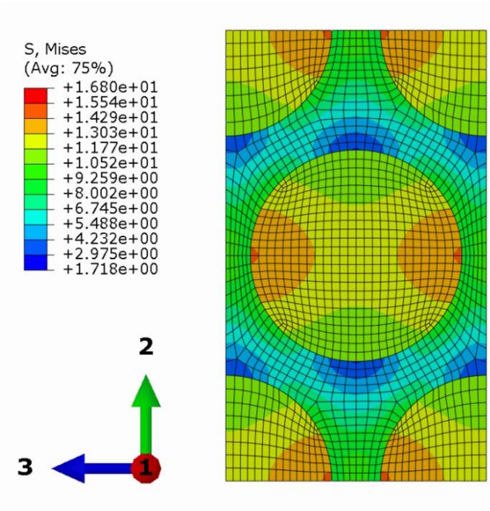
K. A. Malusare and R. S. Fertig, American Institute of Aeronautics and Astronautics, 2014.

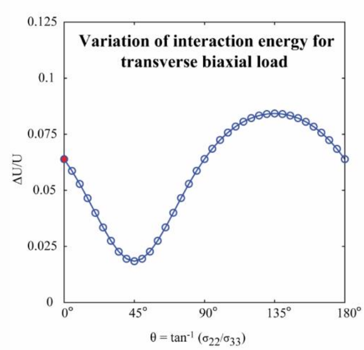
Effect of biaxial loading on interaction energy

- Five types of biaxial loads were considered
- $\sigma_{22} - \sigma_{33}$
- $\sigma_{12} - \sigma_{22}$
- $\sigma_{12} - \sigma_{23}$
- $\sigma_{12} - \sigma_{13}$
- $\sigma_{23} - \sigma_{22}$
- Biaxial load represented by radius of circle
- θ is varied from 0 ° to 180°



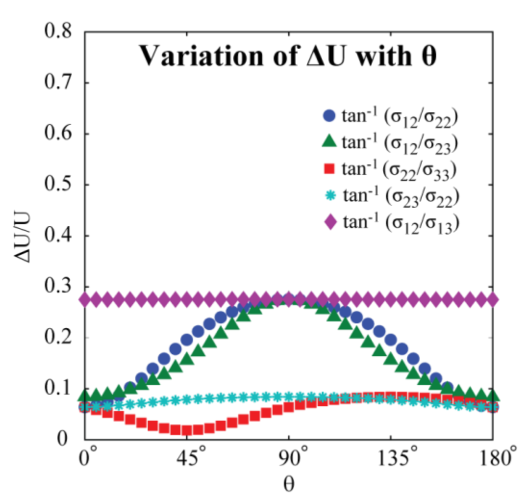
Effect of transverse biaxial loading on interaction energy





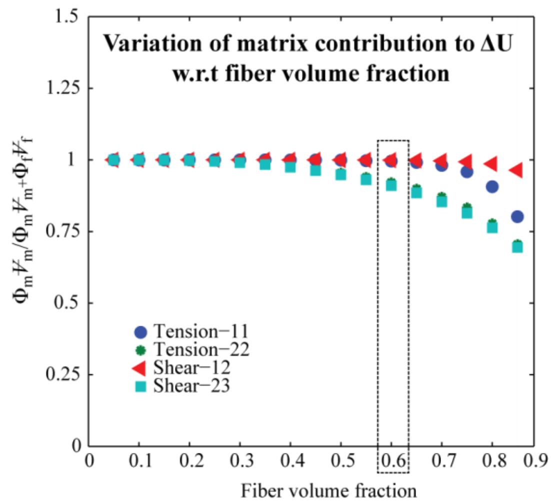
Effect of biaxial loading on interaction energy

- Fiber material is carbon
  $E_m = 1.702\% \ E_f$
  $= 4.0 \ GPa$
- For $\sigma_{12} - \sigma_{13}$ interaction energy is constant
- For $\sigma_{22} - \sigma_{33}$ interaction energy is minimum at 45° and peaks at 135°
- For remaining three cases interaction energy is maximum at an angle of 90°



K. A. Malusare and R. S. Fertig, American Institute of Aeronautics and Astronautics, 2014.

Matrix contribution to interaction energy



K. A. Malusare and R. S. Fertig, American Institute of Aeronautics and Astronautics, 2014.

Challenges

- Interaction energy is in the range of 30-40% of total energy for shear loading for carbon-epoxy systems.
- All this interaction energy is due to the matrix
- Can we augment the matrix stresses with the interaction energy to improve failure load predictions ?

Overview

1. Failure modeling techniques - HOW to predict failure ?

PROBLEM

2. Missing strain energy - "Interaction energy"
3. Investigate the nature of "Interaction energy" using FEA
4. **Failure modeling using volume average constituent stresses**

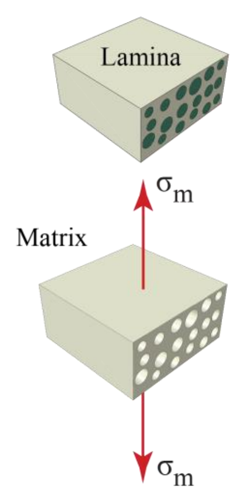
SOLUTION

5. Modeling fluctuations and augmenting volume average quantities
6. Failure modeling using energy conserving constituent stresses

7. Summary/Conclusions
8. Future work

Fertig matrix failure theory[14]

$$B_t\{I_t\}^2 + \frac{1}{1+\frac{\beta}{\tau_0}\{-I_h\}}\left[B_{s1}I_{s1} + B_{s2}I_{s2}\right] = 1$$



$$I_t = \frac{\sigma_{22}^m + \sigma_{33}^m + \sqrt{\left(\sigma_{22}^m + \sigma_{33}^m\right)^2 - 4\left(\sigma_{22}^m\sigma_{33}^m - \sigma_{23}^m\sigma_{23}^m\right)}}{2} \quad \text{(Tension)}$$

$$I_{s1} = \sigma_{12}^{m\,2} + \sigma_{13}^{m\,2} \quad \text{(Longitudinal Shear)}$$

$$I_{s2} = \frac{1}{4}\left(\sigma_{22}^m - \sigma_{33}^m\right)^2 + \sigma_{23}^{m\,2} \quad \text{(Transverse Shear)}$$

$$I_h = \sigma_{22}^m + \sigma_{33}^m \quad \text{(Effect of pressure on shear plane)}$$

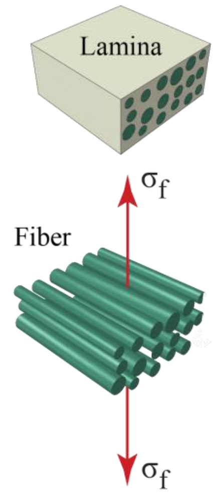
14. Ray S. Fertig, III, "Bridging the gap between physics and large-scale structural analysis: a novel method for fatigue life prediction of composites", *SAMPE 2009 Fall Technical Conference – Wichita, KS, October 19-22, 2009.*

Fiber failure theory

The Maximum stress failure criterion used

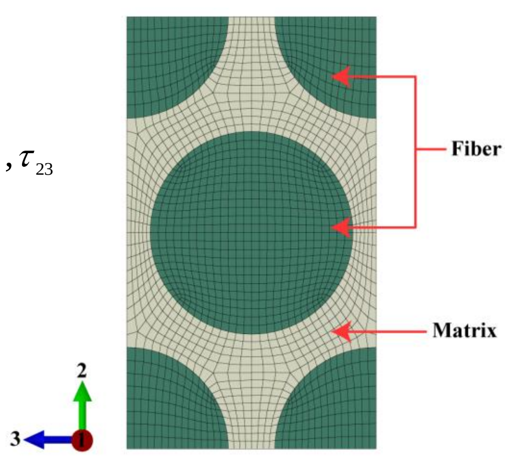
$$\frac{\sigma_{11}^{f}}{S_{11}^{f+}}=1 \quad \text{or} \quad \frac{\sigma_{11}^{f}}{S_{11}^{f-}}=1$$

S_{11}^{f+} longitudinal tensile strength of the fiber
S_{11}^{f-} compressive strength of the fiber



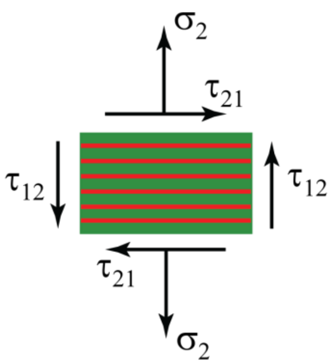
Computing constituent stresses

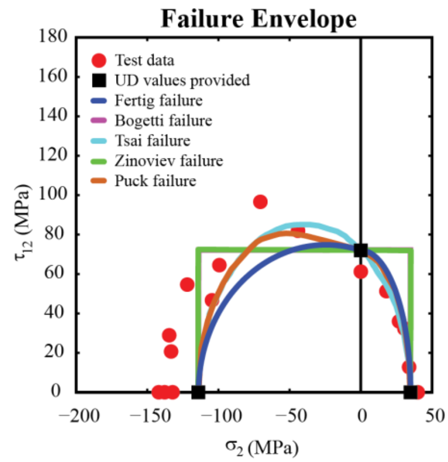
- RVE with hexagonal fiber packing
- Loads $\sigma_{11},\sigma_{22},\sigma_{33},\tau_{12},\tau_{13},\tau_{23}$
- Mapping ${}_{L}\mathrm{X}_{i}^{a}=\frac{\sigma_{i}^{a}}{\sigma_{L}^{C}}$
- Can obtain constituent stresses for any composite stress state



K. A. Malusare and R. S. Fertig, American Institute of Aeronautics and Astronautics, 2014.

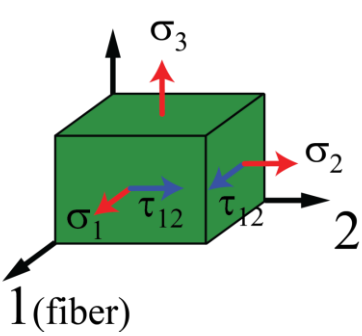
GRP lamina under combined transverse and shear loading

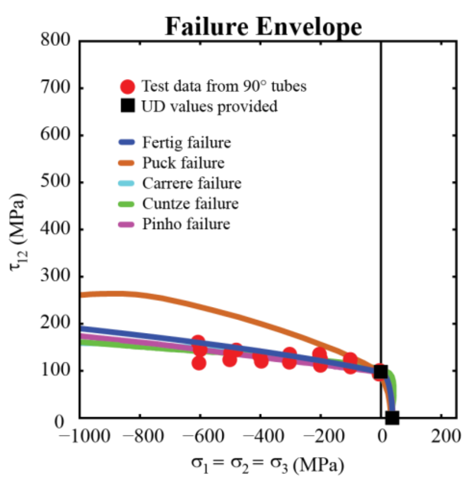




K. A. Malusare and R. S. Fertig, International Conference on Future Technologies for Wind Energy, Laramie , 2013.

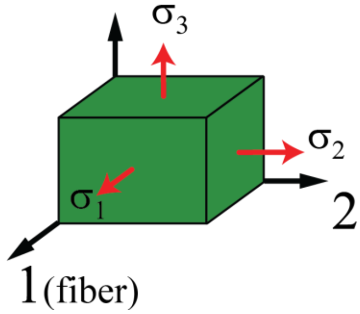
CFRP lamina under combined hyrdostatic and shear loading

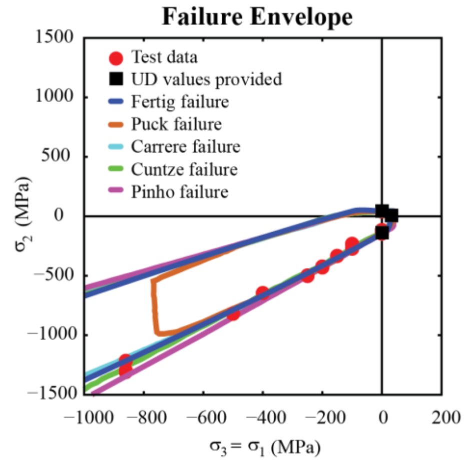




K. A. Malusare and R. S. Fertig, International Conference on Future Technologies for Wind Energy, Laramie , 2013.

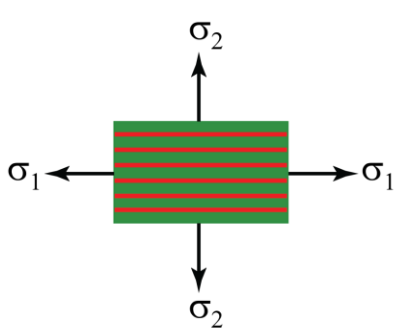
GRP lamina under combined transverse and through thickness loading

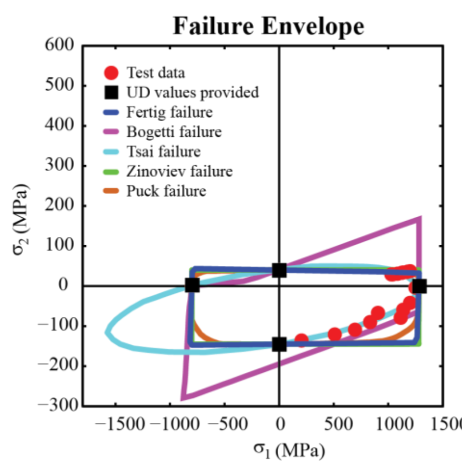




K. A. Malusare and R. S. Fertig, International Conference on Future Technologies for Wind Energy, Laramie , 2013.

GRP lamina under combined longitudinal and transverse loading





K. A. Malusare and R. S. Fertig, International Conference on Future Technologies for Wind Energy, Laramie , 2013.

The von Mises-maximum principal stress failure theory

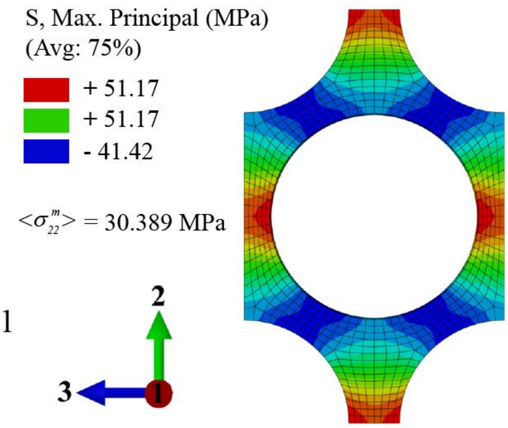
- RVE must be subjected to transverse failure load.

$$\alpha_m = \frac{\sigma^m_{maximum\ principal}}{<\sigma^m_{22}>}$$

- Matrix failure theory is

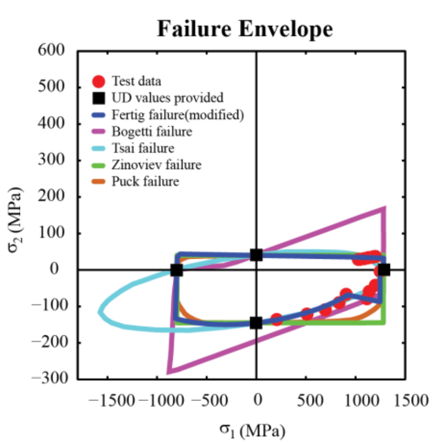
$$\frac{\sigma^m_{maximum\ principal}}{S^m_{+t}\ \alpha_m} = 1 \quad \text{or} \quad \frac{\sigma^m_{VM}}{S^m_{VM}} = 1$$

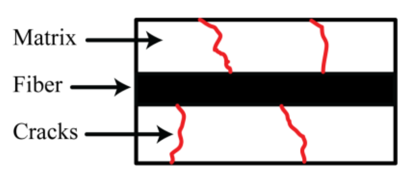
- Fiber failure criterion is same.

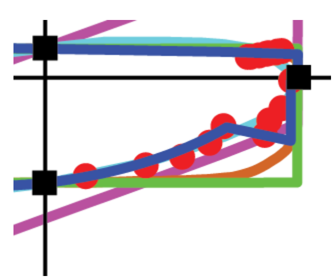


K. A. Malusare and R. S. Fertig, International Conference on Future Technologies for Wind Energy, Laramie , 2013.

GRP lamina under combined longitudinal and transverse loading







K. A. Malusare and R. S. Fertig, International Conference on Future Technologies for Wind Energy, Laramie , 2013.

Overview

1. Failure modeling techniques - HOW to predict failure ?

PROBLEM

2. Missing strain energy - "Interaction energy"
3. Investigate the nature of "Interaction energy" using FEA
4. Failure modeling using volume average constituent stresses

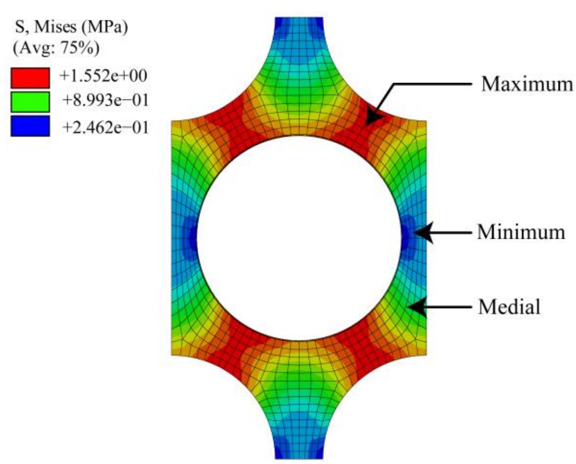
SOLUTION

5. Modeling fluctuations and augmenting volume average quantities
6. Failure modeling using energy conserving constituent stresses

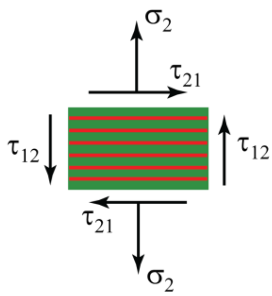
7. Summary/Conclusions
8. Future work

Modeling fluctuations

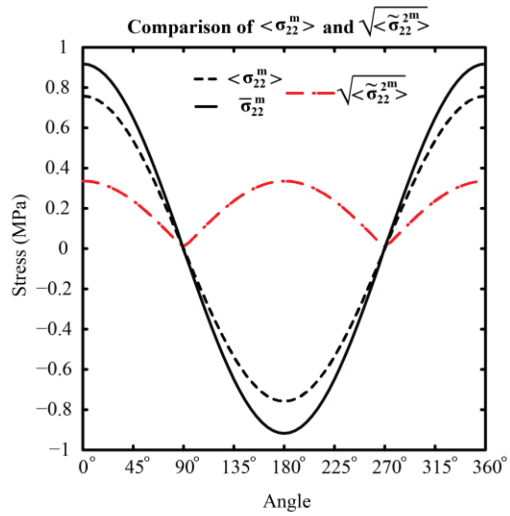
- Used an RVE with Hexagonal fiber packing with fiber VF - 0.6
- Subjected to ***unit*** biaxial/tri-axial loading
- Stress/strain fluctuations of matrix constituent were extracted
- Two types of matrix fluctuations were observed



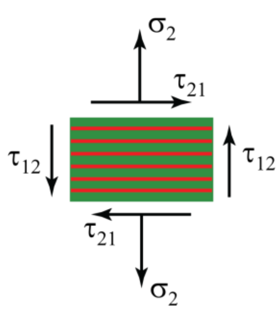
Modeling fluctuations - Type 1



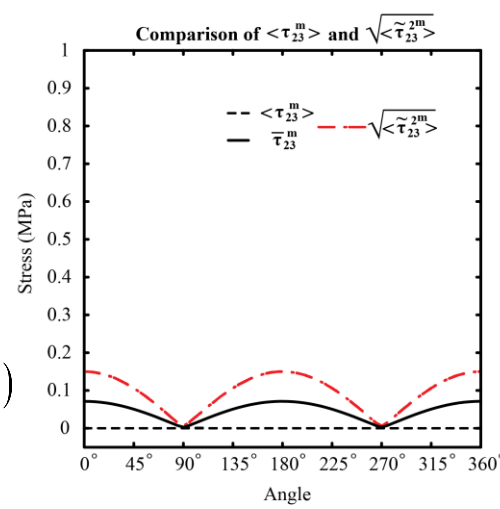
$$\sqrt{\left\langle \tilde{\sigma}_{ij}^{m\,2} \right\rangle} = F_{ij} \left\langle \sigma_{ij}^{m} \right\rangle$$



Modeling fluctuations - Type 2



$$\sqrt{\left\langle \tilde{\sigma}_{ij}^{m\,2} \right\rangle} = L\left(A_{ij}\left|\cos\theta\right| + B_{ij}\right)$$



Interaction energy due to energy conserving quantities

- Strain energy of composite

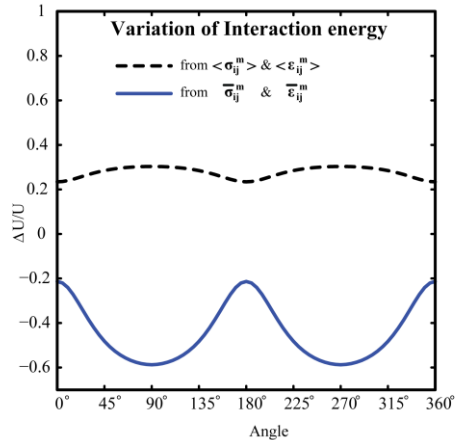
$$U = \frac{1}{2}\sigma_{ij}^{c}\varepsilon_{ij}^{c}V_f$$

- Strain energy of composite from constituents

$$U_{new} = \frac{1}{2}\sigma_{ij}^{f}\varepsilon_{ij}^{f}V_f + \frac{1}{2}\bar{\sigma}_{ij}^{m}\bar{\varepsilon}_{ij}^{m}V_f$$

- Comparison of strain energies

$$U > U_{new} \qquad \Delta U = U - U_{new}$$



41

UNIVERSITY of WYOMING

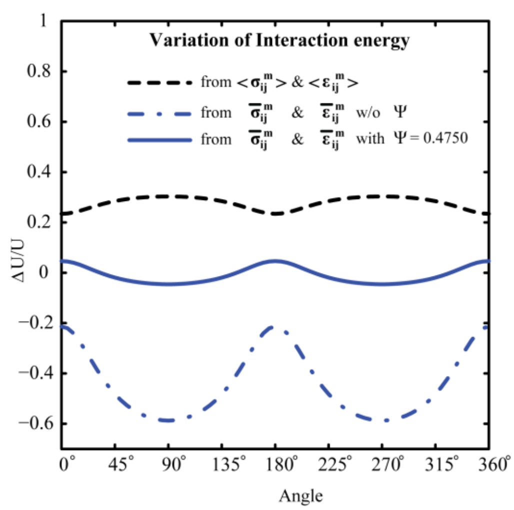
Augmenting quantities

- Need to incorporate the fluctuation energy constant - Ψ

$$\bar{\sigma}_{ij}^{m} = \pm\langle\sigma_{ij}^{m}\rangle \pm \Psi\sqrt{\langle\tilde{\sigma}_{ij}^{m\,2}\rangle}$$

$$\bar{\varepsilon}_{ij}^{m} = \pm\langle\varepsilon_{ij}^{m}\rangle \pm \Psi\sqrt{\langle\tilde{\varepsilon}_{ij}^{m\,2}\rangle}$$

- Range of Psi is $0 \le \Psi \le 1$
- Psi is obtained by iteration
- Psi depends on material properties, configuration of loading & type of fiber packing



42

UNIVERSITY of WYOMING

Overview

1. Failure modeling techniques - HOW to predict failure ?

PROBLEM

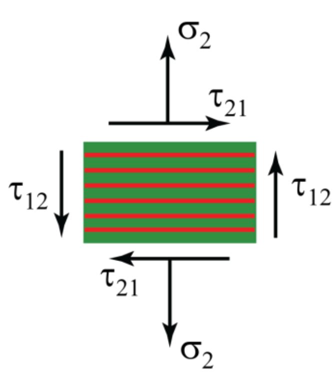
2. Missing strain energy - "Interaction energy"
3. Investigate the nature of "Interaction energy" using FEA
4. Failure modeling using volume average constituent stresses

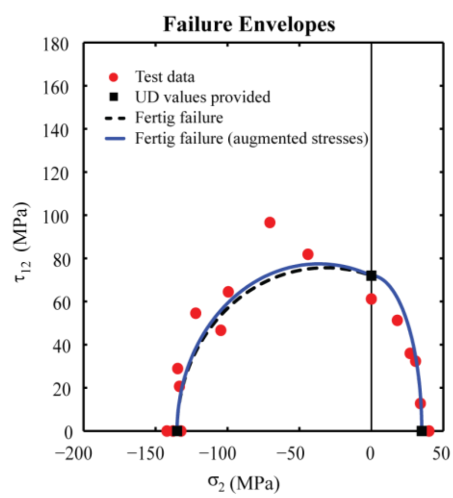
SOLUTION

5. Modeling fluctuations and augmenting volume average quantities
6. Failure modeling using energy conserving constituent stresses

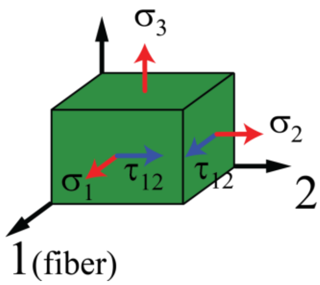
7. Summary/Conclusions
8. Future work

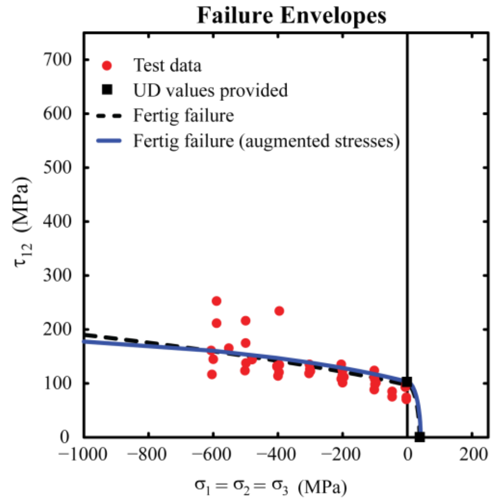
GRP lamina under combined transverse and shear loading



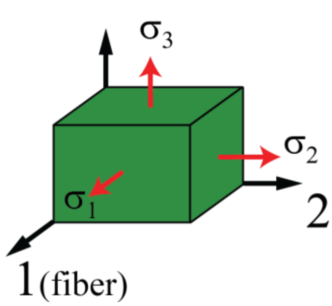


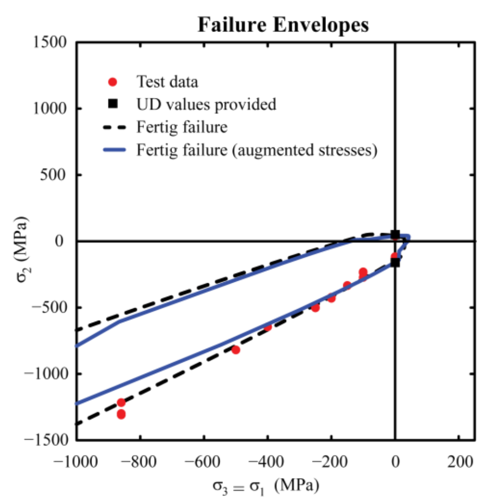
CFRP lamina under combined hyrdostatic and shear loading



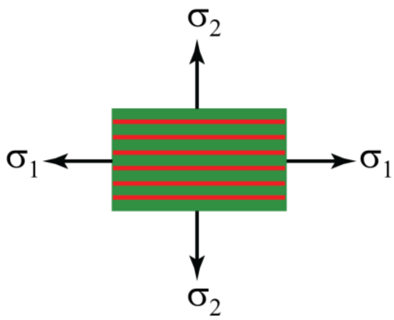


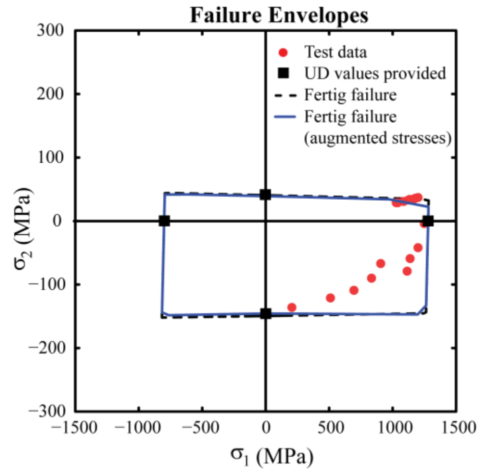
GRP lamina under combined transverse and through thickness loading





GRP lamina under combined longitudinal and transverse loading





Overview

1. Failure modeling techniques - HOW to predict failure ?

PROBLEM

2. Missing strain energy - "Interaction energy"
3. Investigate the nature of "Interaction energy" using FEA
4. Failure modeling using volume average constituent stresses

SOLUTION

5. Modeling fluctuations and augmenting volume average quantities
6. Failure modeling using energy conserving constituent stresses

7. Summary/Conclusions
8. Future work

Summary

- Stress/strain fluctuations in the constituents give rise to Interaction energy which can reach 30%
- Interaction energy is mainly due to the fluctuations in the matrix constituent
- Stress/strain fluctuations were extracted from the matrix constituents and the matrix quantities were augmented to minimize interaction energy
- A three parameter micromechanics based Fertig failure theory was used along with energy consistent stresses to predict failure

Conclusions

- The augmented matrix quantities are now energy conserving
- Use of energy conserving matrix stresses improved failure predictions slightly
- Slight improvement in static failure prediction will improve creep and fatigue load predictions significantly.

Overview

1. Failure modeling techniques - HOW to predict failure ?

PROBLEM
2. Missing strain energy - "Interaction energy"
3. Investigate the nature of "Interaction energy" using FEA
4. Failure modeling using volume average constituent stresses

SOLUTION
5. Modeling fluctuations and augmenting volume average quantities
6. Failure modeling using energy conserving constituent stresses

7. Summary/Conclusions

8. Future work

Future work

- Failure envelopes for multiply laminates need to be obtained
- Fertig failure theory needs to be augmented with matrix stresses in the longitudinal direction (σ_{11}^{m})
- Augmented stresses maybe used with other micromechanical theory to see if there is an improvement in failure load predictions

UNIVERSITY OF WYOMING

Thank you
QUESTIONS ?

A closer look at the expression for IE

- Material inhomogeneity increases and decreases with fiber VF

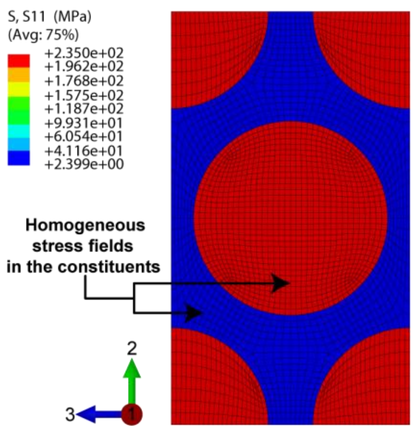
$$\Phi = \frac{1}{2}\left[\begin{array}{l} C_{11}\left\langle(\tilde{\varepsilon}_{11})^2\right\rangle + C_{22}\left\langle(\tilde{\varepsilon}_{22})^2\right\rangle + C_{33}\left\langle(\tilde{\varepsilon}_{33})^2\right\rangle + 2C_{12}\langle\tilde{\varepsilon}_{11}\cdot\tilde{\varepsilon}_{22}\rangle + 2C_{13}\langle\tilde{\varepsilon}_{11}\cdot\tilde{\varepsilon}_{33}\rangle + \\ 2C_{23}\langle\tilde{\varepsilon}_{22}\cdot\tilde{\varepsilon}_{33}\rangle + C_{12}\left\langle(\tilde{\gamma}_{12})^2\right\rangle + C_{13}\left\langle(\tilde{\gamma}_{13})^2\right\rangle + C_{23}\left\langle(\tilde{\gamma}_{23})^2\right\rangle \end{array}\right]$$

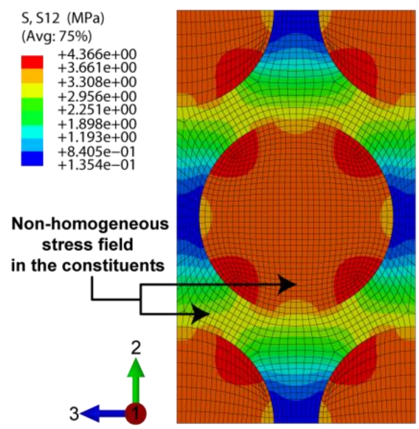
- $\Phi = f(\tilde{\varepsilon}) = f(\tilde{\sigma})$ <u>and</u> $\Delta U = \Phi_f V_f + \Phi_m V_m$
- <u>So</u> $\Delta U = f(\tilde{\varepsilon}) = f(\tilde{\sigma})$

Q1 : Negligible $\tilde{\varepsilon}/\tilde{\sigma}$ in Tension-11 Q2 : Maximum $\tilde{\varepsilon}/\tilde{\sigma}$ in Shear-12

UNIVERSITY OF WYOMING

Distribution of stress with load case for fiber VF 0.6





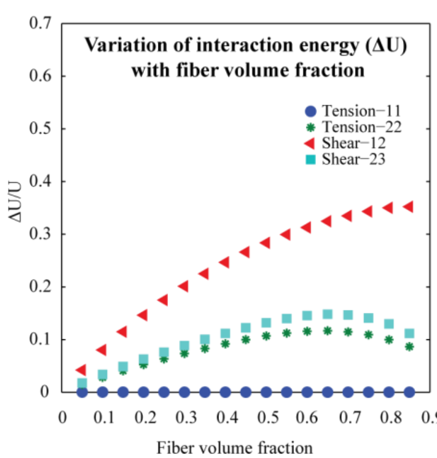
ΔU in Tension-11 is negligible

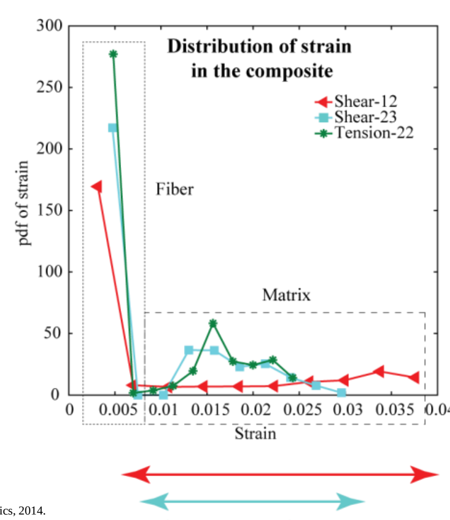
ΔU in Shear-12 is about 30%

K. A. Malusare and R. S. Fertig, American Institute of Aeronautics and Astronautics, 2014.

UNIVERSITY OF WYOMING

Interaction energy in tension-22 and shear-23

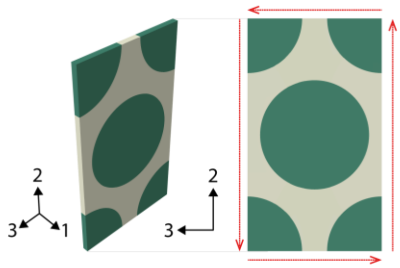




K. A. Malusare and R. S. Fertig, American Institute of Aeronautics and Astronautics, 2014.

UNIVERSITY OF WYOMING

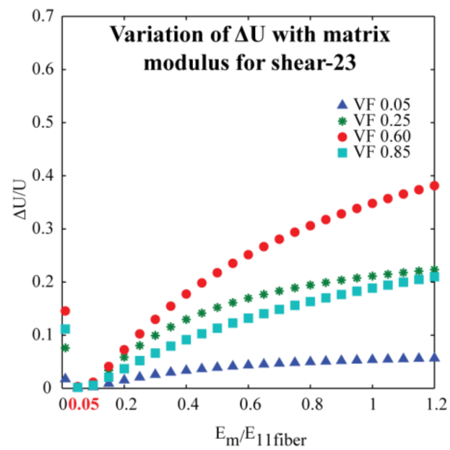
Effect of material properties on interaction energy



$$E_{22_{matrix}} = 5\% \, E_{11_{fiber}} = 11.75GPa$$

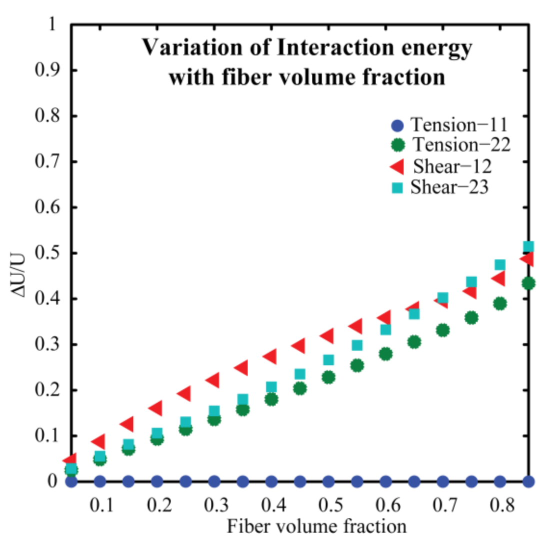
$$G_{23_{matrix}} = 4.38 \; GPa$$
$$G_{23_{fiber}} = 5.60 \; GPa$$

Interaction Energy is minimum

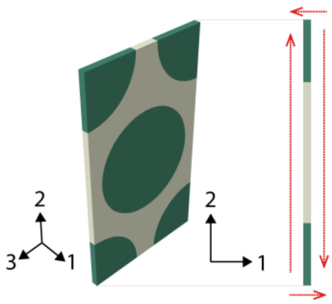


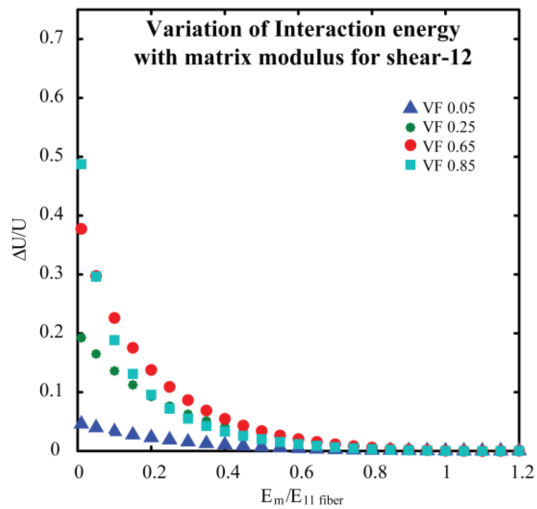
K. A. Malusare and R. S. Fertig, American Institute of Aeronautics and Astronautics, 2014.

Effect of fiber volume fraction on interaction energy



Effect of material properties on interaction energy





Material properties

		VF variation	Matrix modulus variation	Biaxial loading
Material	Fiber	Matrix	Matrix	Matrix
Material type	Transversely isotropic	Isotropic	Isotropic	Isotropic
$E_{11}(GPa)$	235.0	$0.01E_{11}$ (2.35)	$0.01E_{11}$ *to* $1.2E_{11}$	4.0
$E_{22}\ (GPa)$	14.0	$0.01E_{11}$ (2.35)	$0.01E_{11}$ *to* $1.2E_{11}$	4.0
$G_{12}(GPa)$	28.0	0.8769	Varies with matrix modulus	1.493
ν_{12}	0.2	0.34	0.34	0.34
ν_{23}	0.25	0.34	0.34	0.34

UNIVERSITY OF WYOMING

Material properties WWFE-1

Fibre type	AS4	T300	E-glass 21xK43 Gevetex	Silenka E-Glass 1200tex
Matrix	3501-6 epoxy	BSL914C epoxy	LY556/HT907/ DY063 epoxy	MY750/HY917/ DY063 epoxy
Specification	Prepeg	Filament winding	Filament winding	Filament winding
Manufacturer	Hercules	DFVLR	DLR	DRA
Fibre volume fraction, V_f	0·60	0·60	0·62	0·60
Longitudinal modulus, E_1 (GPa)	126[a]	138	53·48	45·6
Transverse modulus, E_2 (GPa)	11	11	17·7	16·2
In-plane shear modulus, G_{12} (GPa)	6·6[a]	5·5[a]	5·83[a]	5·83[a]
Major Poisson's ratio, ν_{12}	0·28	0. 28	0·278	0·278
Through thickness Poisson's ratio, ν_{23}	0·4	0·4	0·4	0·4
Longitudinal tensile strength, X_T (MPa)	1950[b]	1500	1140	1280
Longitudinal compressive strength, X_c (MPa)	1480	900	570	800
Transverse tensile strength, Y_T (MPa)	48	27	35	40
Transverse compressive strength, Y_c (MPa)	200[b]	200	114	145[b]
In-plane shear strength, S_{12} (MPa)	79[b]	80[b]	72[b]	73[b]
Longitudinal tensile failure strain, ε_{1T} (%)	1·38	1·087	2·132	2·807
Longitudinal compressive failure strain ε_{1C} (%)	1·175	0·652	1·065	1·754
Transverse tensile failure strain ε_{2T} (%)	0·436	0·245	0·197	0·246
Transverse compressive failure strain, ε_{2C} (%)	2·0	1·818	0·644	1·2
In-plane shear failure strain, γ_{12u} (%)	2	4	3·8	4
Strain energy release rate, G_{IC} (J m^{-2})	220	220	165	165
Longitudinal thermal coefficient, α_1 (10^{-6}/°C)	−1	−1	8·6	8·6
Transverse thermal coefficient, α_2 (10^{-6}/°C)	26	26	26·4	26·4
Stress free temperature (°C)	177	120	120	120
Curing			2 h at 120°C 2 h at 150°C	2 h at 90°C 1·5 h at 130°C 2 h at 150°C

[a]Initial modulus.
[b]Nonlinear behaviour and stress/strain curves and data points are provided.

Material properties WWFE-1

Fibre type	AS4	T300	E-glass 21xK43 Gevetex	Silenka E-Glass 1200tex
Longitudinal modulus, E_{f1} (GPa)	225	230	80	74
Transverse modulus, E_{f2} (GPa)	15	15	80	74
In-plane shear modulus, G_{f12} (GPa)	15	15	33·33	30·8
Major Poisson's ratio, ν_{f12}	0·2	0·2	0·2	0·2
Transverse shear modulus, G_{f23}	7	7	33·33	30·8
Longitudinal tensile strength, X_{fT} (MPa)	3350	2500	2150	2150
Longitudinal compressive strength, X_{fc} (MPa)	2500	2000	1450	1450
Longitudinal tensile failure strain, ε_{f1T} (%)	1·488	1·086	2·687	2·905
Longitudinal compressive failure strain, ε_{f1C} (%)	1·111	0·869	1·813	1·959
Longitudinal thermal coefficient, α_{f1} (10^{-6}/°C)	−0·5	−0·7	4·9	4·9
Transverse thermal coefficient, α_{f2} (10^{-6}/°C)	15	12	4·9	4·9

Matrix type	3501-6 epoxy	BSL914C epoxy	LY556/HT907/ DY063 epoxy	MY750/HY917/ DY063 epoxy
Manufacturer	Hercules	DFVLR	Ciba Geigy	Ciba Geigy
Modulus, E_m (Gpa)	4·2	4·0	3·35	3.35
Shear modulus, G_m (Gpa)	1·567	1·481	1·24	1·24
Poisson's ratio, ν_m	0·34	0·35	0·35	0·35
Tensile strength, Y_{mT} (MPa)	69	75	80	80
Compressive strength, Y_{mC} (MPa)	250	150	120	120
Shear strength, S_m (MPa)	50	70	—	—
Tensile failure strain, ε_{mT} (%)	1·7	4	5	5
Thermal coefficient, α_m (10^{-6}/°C)	45	55	58	58

Material properties WWFE-2

Fibre type	IM7	T300	A-S	S2-glass	E-Glass
Matrix	8551-7	PR-319	Epoxy 1	Epoxy 2	MY750
Fibre volume fraction V_f (%)	60	60	60	60	60
Longitudinal modulus E_1 (GPa)	165[a]	129	140[a]	52	45.6
Transverse modulus E_2 (GPa)	8.4	5.6[c]	10	19	16.2
Through-thickness modulus E_3 (GPa)	8.4	5.6[c]	10	19	16.2
In-plane shear modulus G_{12} (GPa)	5.6[a]	1.33[c]	6[a]	6.7[a]	5.83[a]
Transverse shear modulus G_{13} (GPa)	5.6[a]	1.33[c]	6[a]	6.7[a]	5.83[a]
Through-thickness shear modulus G_{23} (GPa)	2.8	1.86	3.35	6.7	5.7
Major Poisson's ratio υ_{12}	0.34	0.318	0.3	0.3	0.278
Major transverse Poisson's ratio υ_{13}	0.34	0.318	0.3	0.3	0.278
Through-thickness Poisson's ratio υ_{23}	0.5	0.5	0.49	0.42	0.4
Longitudinal tensile strength X_T (MPa)	2560	1378	1990	1700	1280
Longitudinal compressive strength X_C (MPa)	1590	950	1500	1150	800
Transverse tensile strength Y_T (MPa)	73	40	38	63	40
Transverse compressive strength Y_C (MPa)	185[b]	125[b]	150[b]	180[b]	145[b]
Through-thickness tensile strength Z_T (MPa)	63	40	38	50	40
Through-thickness compressive strength Z_C (MPa)	185[b]	125[b]	150[b]	180[b]	145[b]
In-plane shear strength S_{12} (MPa)	90[b]	97[b]	70[b]	72[b]	73[b]
Transverse shear strength S_{13} (MPa)	90[b]	97[b]	70[b]	72[b]	73[b]
Through-thickness shear strength S_{23} (MPa)	57	45	50	40	50
Longitudinal tensile failure strain ε_{1T} (%)	1.551	1.07	1.42	3.27	2.807
Longitudinal compressive failure strain ε_{1C} (%)	1.1	0.74	1.2	2.21	1.754
Transverse tensile failure strain ε_{2T} (%)	0.87	0.43	0.38	0.33	0.246
Transverse compressive failure strain ε_{2C} (%)	3.2	2.8	1.6	1.5	1.2
Through-thickness tensile failure strain ε_{3T} (%)	0.755	0.43	0.38	0.263	0.246
Through-thickness compressive failure strain ε_{3C} (%)	3.2	2.8	1.6	1.5	1.2
In-plane shear failure strain γ_{12u} (%)	5	8.6	3.5	4	4
Transverse shear failure strain γ_{13u} (%)	5	8.6	3.5	4	4
Through-thickness shear failure strain γ_{23u} (%)	2.1	1.5	1.5	0.59	0.88
Longitudinal thermal coefficient α_1 (10^{-6}/°C)	−1	−1	−1	8.6	8.6
Transverse thermal coefficient α_2 (10^{-6}/°C)	18	26	26	26.4	26.4
Through-thickness thermal coefficient α_3 (10^{-6}/°C)	18	26	26	26.4	26.4
Stress free temperature (°C)	177	120	120	120	120

[a] Initial modulus.
[b] Nonlinear behaviour and stress–strain curves and data points are provided.
[c] Please note that values are considered to be low, compared with typical data for the same material published somewhere else or quoted by the manufacturers. We have not attempted to change them in order to facilitate a comparison with test data in Part B of the exercise.

UNIVERSITY OF WYOMING

Material properties WWFE-2

Fibre type	IM7	T300	AS	S2-glass	E-Glass
Longitudinal modulus E_{f1} (GPa)	276	231	231	87	74
Transverse modulus E_{f2} (GPa)	19	15	15	87	74
Transverse modulus E_{f3} (GPa)	19	15	15	87	74
In-plane shear modulus G_{f12} (GPa)	27	15	15	36	30.8
Major Poisson's ratio υ_{f12}	0.2	0.2	0.2	0.2	0.2
Major Poisson's ratio υ_{f13}	0.2	0.2	0.2	0.2	0.2
Transverse shear modulus G_{f23} (GPa)	7	7	7	36	30.8
Longitudinal tensile strength X_{fT} (MPa)	5180	2500	3500	2850	2150
Longitudinal compressive strength X_{fC} (MPa)	3200	2000	3000	2450	1450
Longitudinal tensile failure strain ε_{f1T} (%)	1.87	1.086	1.515	3.27	2.905
Longitudinal compressive failure strain ε_{f1C} (%)	1.16	0.869	1.298	2.82	1.959
Longitudinal thermal coefficient α_{f1} (10^{-6}/°C)	−0.4	−0.7	−0.7	5	4.9
Transverse thermal coefficient α_{f2} (10^{-6}/°C)	5.6	12	12	5	4.9
Through-thickness thermal coefficient α_{f3} (10^{-6}/°C)	5.6	12	12	5	4.9

Matrix type	8551-7 epoxy	PR319 epoxy	Epoxy 1	Epoxy 2	MY750
Elastic modulus E_m (GPa)	4.08	0.95[a]	3.2	3.2	3.35
Elastic shear modulus G_m (GPa)	1.478	0.35[a]	1.2	1.2	1.24
Elastic Poisson's ratio υ_m	0.38	0.35	0.35	0.35	0.35
Tensile strength Y_{mT} (MPa)	99	70	85	73	80
Compressive strength Y_{mC} (MPa)	130	130	120	120	120
Shear strength S_m (MPa)	57	41	50	52	54
Tensile failure strain ε_{mT} (%)	4.4	7.3	2.65	2.5	2.7
Compressive failure strain ε_{mC} (%)	9	13.6	3.75	5	5
Shear failure strain γ_m (%)	5.1	11.5	4.16	6	6
Thermal expansion coefficient α_m (10^{-6}/°C)	46.7	60	58	58	58

[a] These values are considered to be low, compared with typical data for the same material published somewhere else or quoted by the manufacturers. We have not attempted to change them in order to facilitate a comparison with test data in Part B of the exercise.
The behaviour of materials PR319 and Epoxy 1 is taken as linear.

UNIVERSITY OF WYOMING

World Wide Failure Exercises

- WWFEs are composite failure benchmarks for GRPs and CFRPs
- Various failure theories were tested against experimental evidence
- Experiments include strength envelopes for laminae and laminates
- stress-strain curves for laminae and laminates
- Only lamina strength envelopes were predicted

UNIVERSITY OF WYOMING

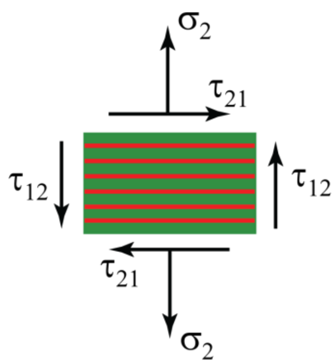
Outcomes of WWFE-1 & WWFE-2

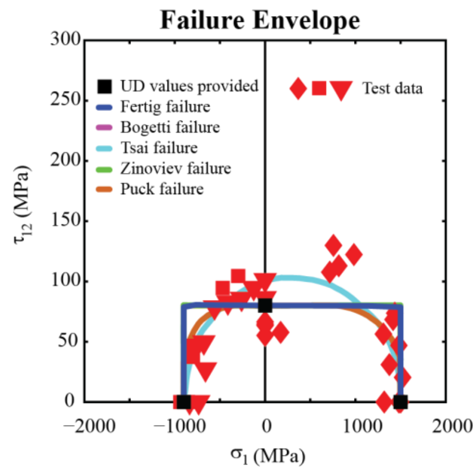
Exercise	Leading theories
WWFE-I	Puck, Zinoviev, Tsai and Bogetti
WWFE-II	Carrere, Pinho, Cuntze and Puck

- Usage of lamina quantities don't permit the use of physics
- Calibration is cumbersome due to large number of input parameters (50-75) parameters

UNIVERSITY OF WYOMING

CFRP lamina under combined transverse and shear loading

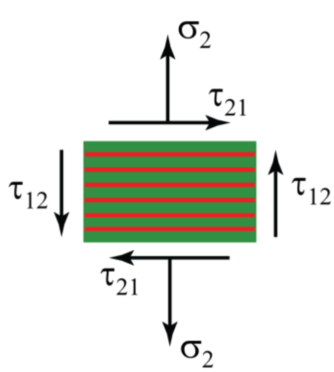


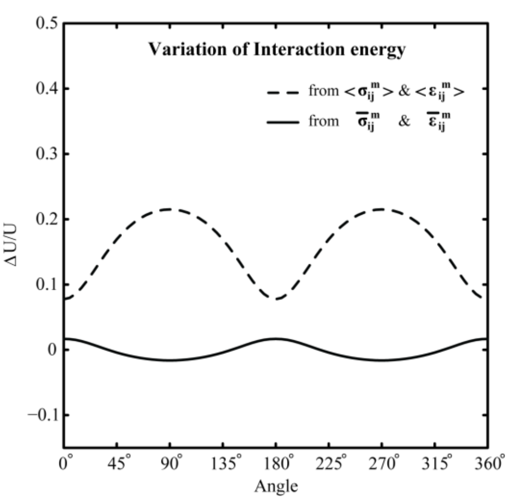


K. A. Malusare and R. S. Fertig, International Conference on Future Technologies for Wind Energy, Laramie , 2013.

UNIVERSITY of WYOMING

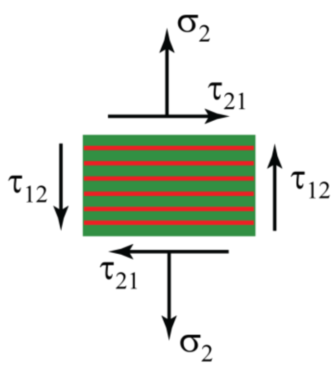
CFRP lamina under combined transverse and shear loading

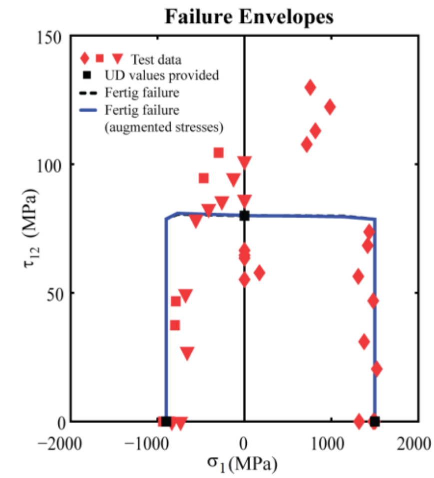




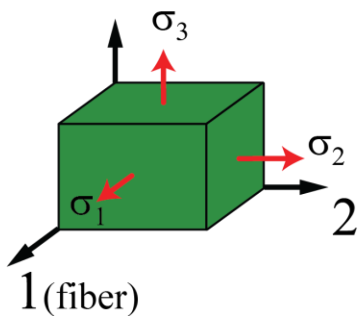
UNIVERSITY of WYOMING

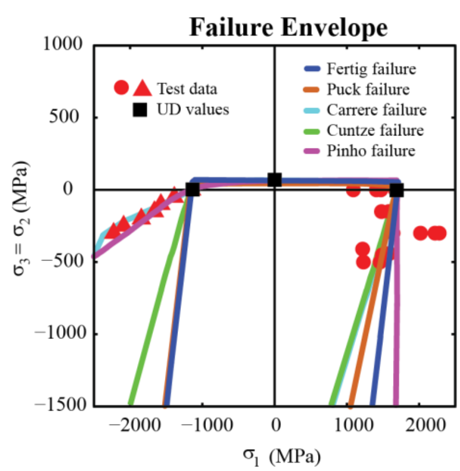
CFRP lamina under combined transverse and shear loading





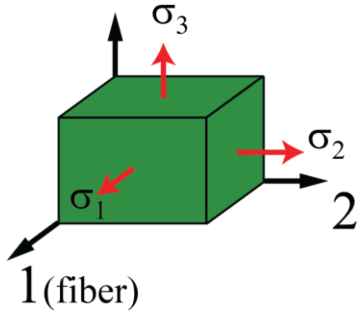
GRP lamina under combined through thickness and longitudinal loading

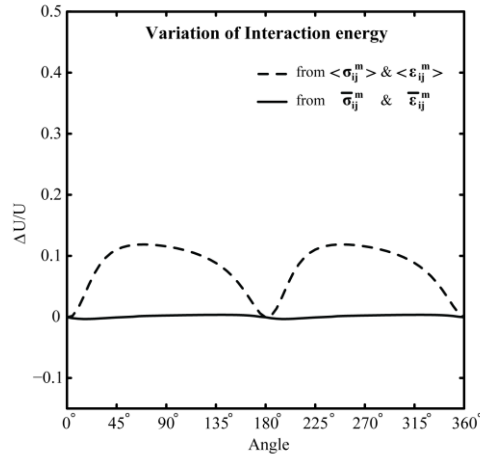




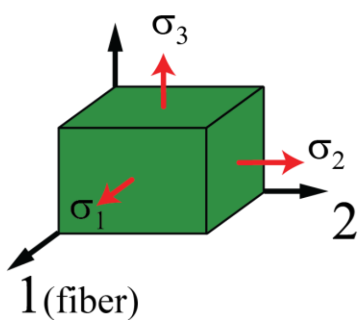
K. A. Malusare and R. S. Fertig, International Conference on Future Technologies for Wind Energy, Laramie , 2013.

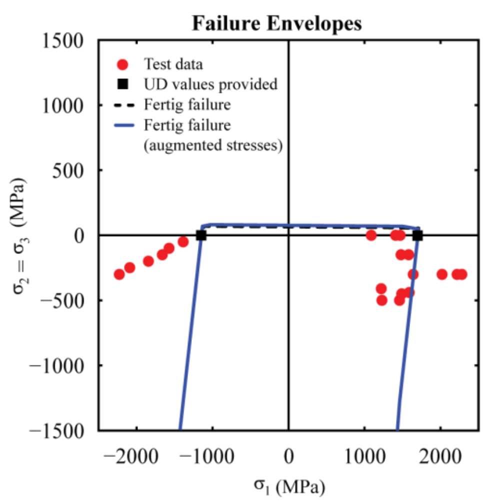
GRP lamina under combined through thickness and longitudinal loading



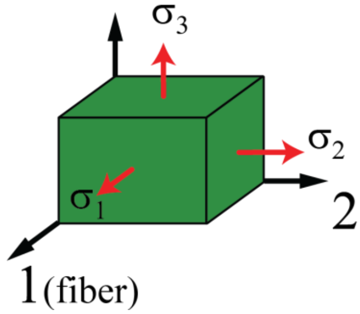


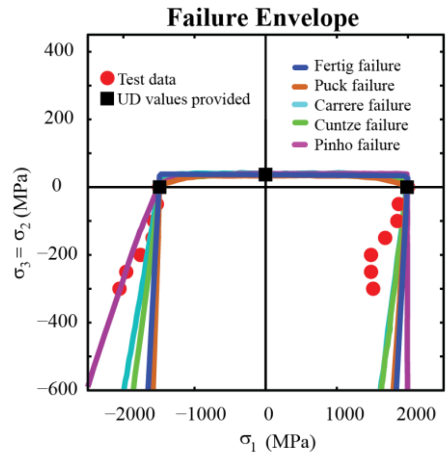
GRP lamina under combined through thickness and longitudinal loading





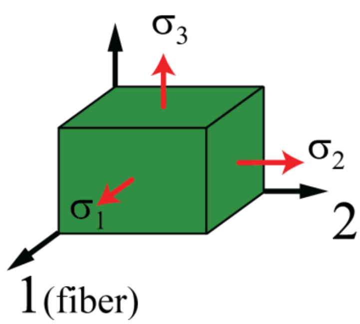
CFRP lamina under combined through thickness and longitudinal loading

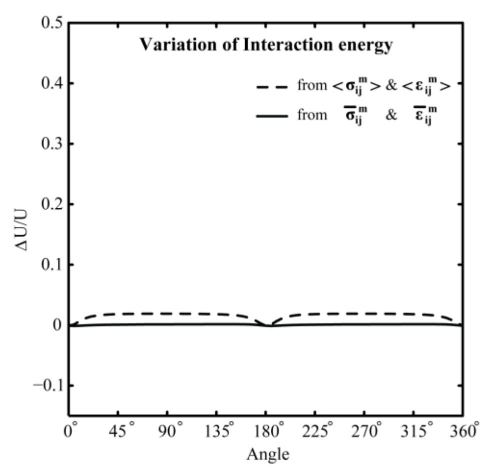




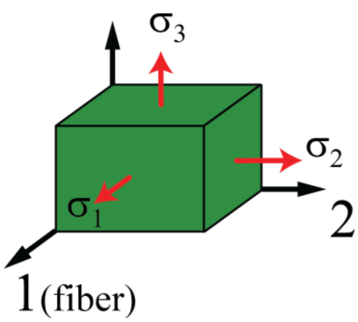
K. A. Malusare and R. S. Fertig, International Conference on Future Technologies for Wind Energy, Laramie , 2013.

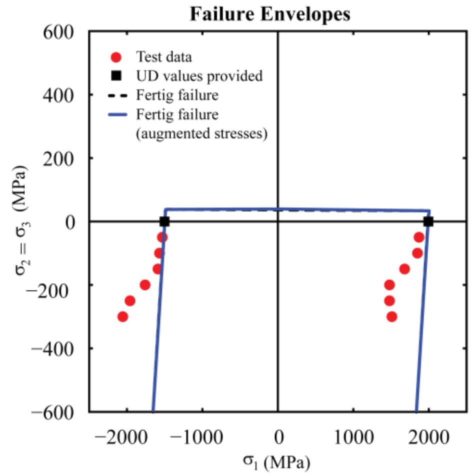
CFRP lamina under combined through thickness and longitudinal loading



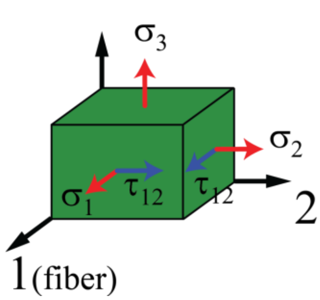


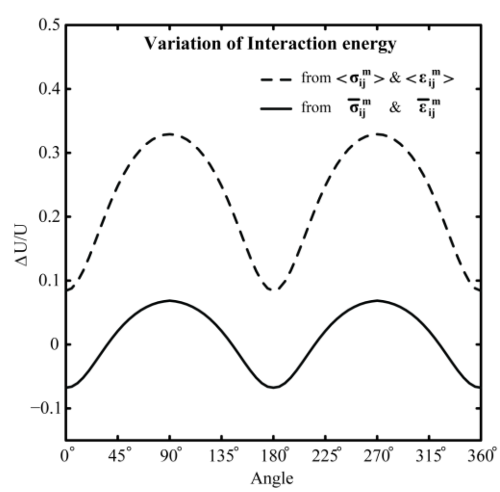
CFRP lamina under combined through thickness and longitudinal loading



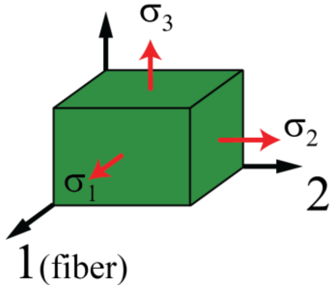


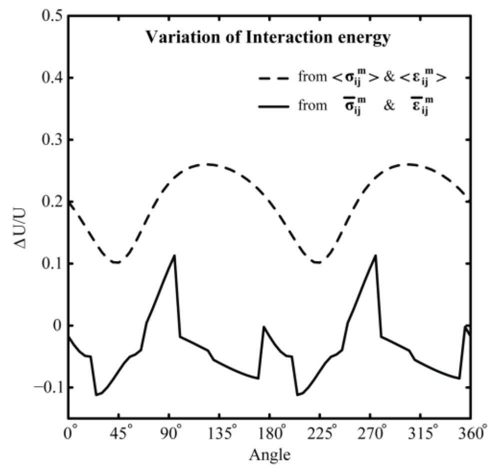
CFRP lamina under combined hyrdostatic and shear loading



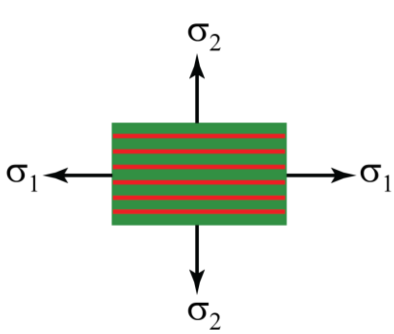


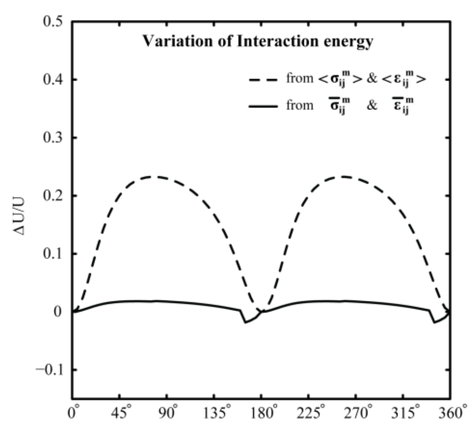
GRP lamina under combined transverse and through thickness loading



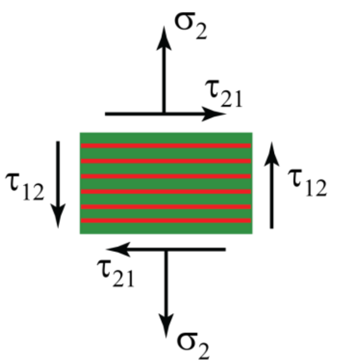


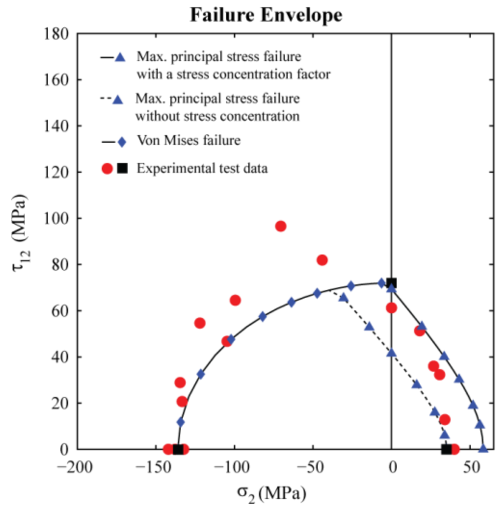
GRP lamina under combined longitudinal and transverse loading



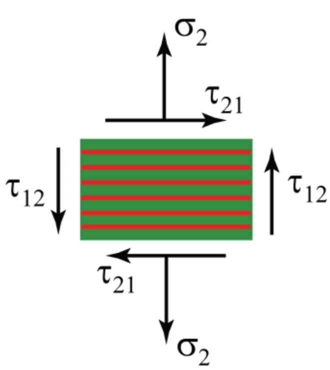


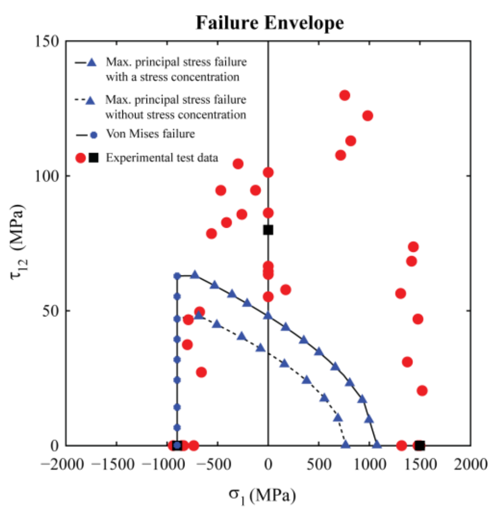
GRP lamina under combined transverse and shear loading



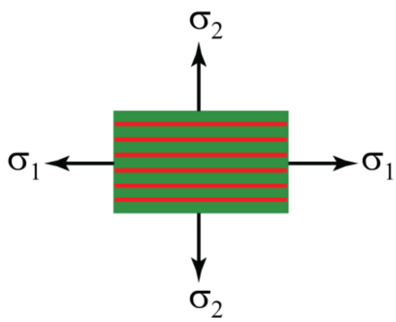


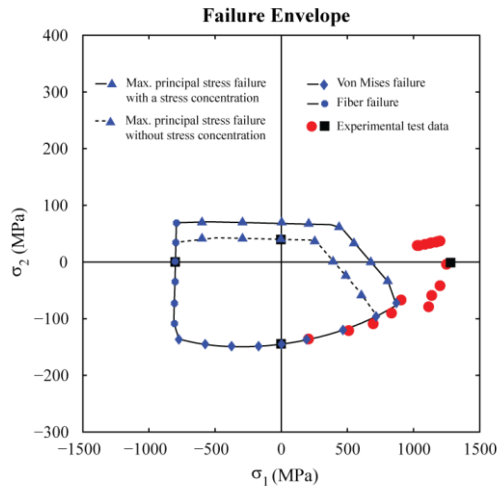
CFRP lamina under combined transverse and shear loading





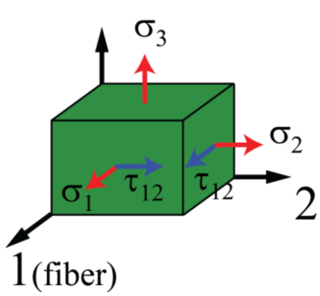
GRP lamina under combined longitudinal and transverse loading

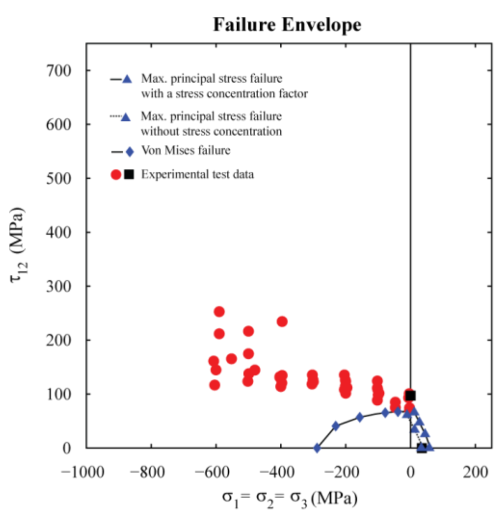




UNIVERSITY OF WYOMING

CFRP lamina under combined hyrdostatic and shear loading

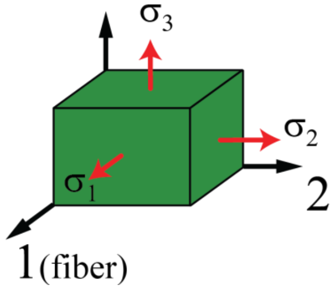


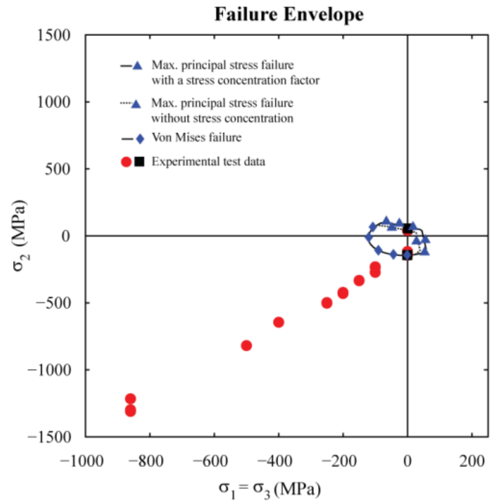


82

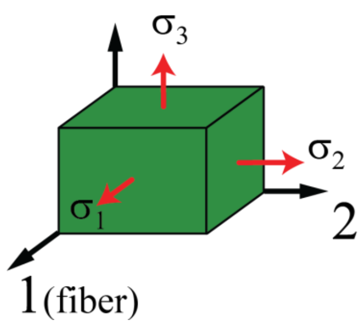
UNIVERSITY OF WYOMING

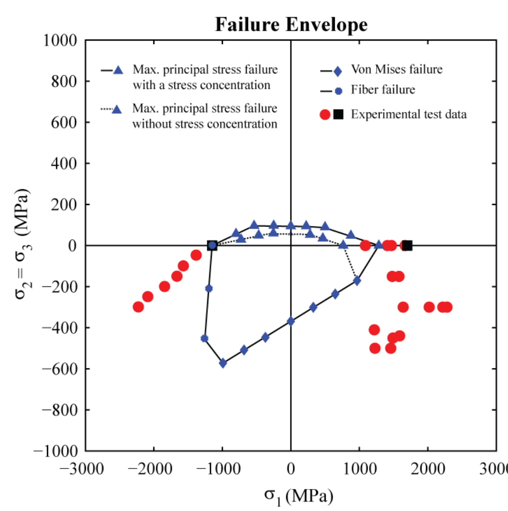
GRP lamina under combined transverse and through thickness loading





GRP lamina under combined through thickness and longitudinal loading





CFRP lamina under combined through thickness and longitudinal loading