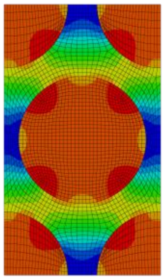
USE OF VOLUME AVERAGE STRESSES TO PREDICT COMPOSITE FAILURE



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54th AIAA/ASME/ASCE/AHS/ ASC Structures, Structural Dynamics, and Materials Conference, Boston 2013.

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Application of composite materials

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Overview

1. Types of failure modeling techniques (Two)
2. Missing strain energy – 'Interaction Energy'
3. FEA model
4. Results of three parametric studies
5. Conclusions

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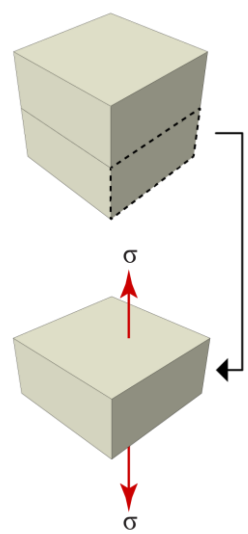
(4/25)

Mesomodeling

- Considers lamina layers (plies) as building blocks of laminates
- Use volume average lamina quantities (stresses & strains) to predict failure
- Examples Maximum stress/strain, Tsai-Wu, Tsai-Hill, Hashin etc.
- Failure prediction remains inadequate

Do lamina quantities capture the true stress/strain state in a constituent ?



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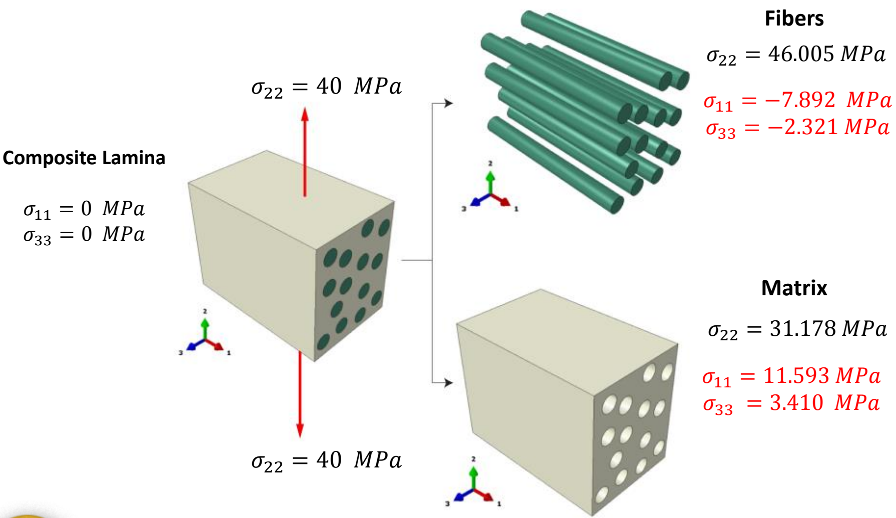
Volume average constituent stresses



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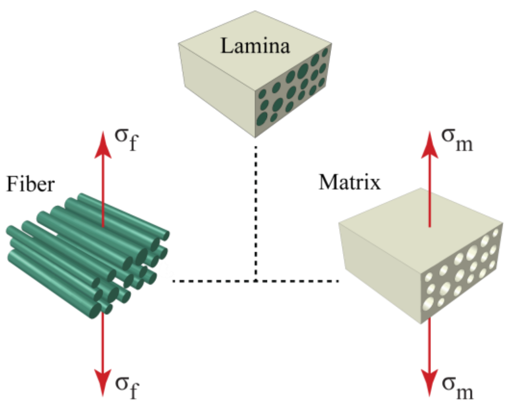
Multiscale micromechanical modeling

- Use average constituent quantities to predict failure
- Can predict the response of the entire composite using just constituent properties
- World Wide Failure Exercise – Chamis, Mayes and Huang

Constituent quantities do not **COMPLETELY** represent the true stress/state in the constituent



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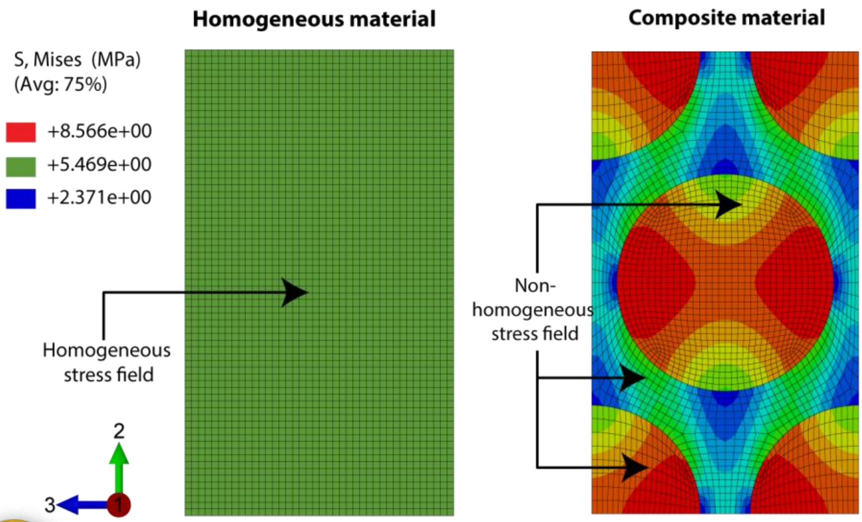
Stress and strain fluctuations



Strain energy comparison

$$U = \frac{1}{2}\,\sigma_{ij}{}^{c}\,\varepsilon_{ij}{}^{c}\,V_c$$

$$U_f = \frac{1}{2}\,\sigma_{ij}{}^{f}\,\varepsilon_{ij}{}^{f}\,V_f \qquad\qquad U_m = \frac{1}{2}\,\sigma_{ij}{}^{m}\,\varepsilon_{ij}{}^{m}\,V_m$$

$$U > U_f + U_m$$

$$U = \left(U_f + U_m\right) + \Delta U$$

where ΔU is the missing energy.

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Interaction energy

$$U_f = \frac{1}{2}\,\sigma_{ij}{}^{f}\,\varepsilon_{ij}{}^{f}\,V_f + \Phi_f V_f$$

$$U_m = \frac{1}{2}\,\sigma_{ij}{}^{m}\,\varepsilon_{ij}{}^{m}\,V_m + \Phi_m V_m$$

$$\Delta U = \Phi_f V_f + \Phi_m V_m$$

where ΔU is the 'Interaction Energy'.

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Interaction energy

$$\Phi_f = \frac{1}{2}\int_{V_f} \tilde{\boldsymbol{\sigma}}_{ij}^{f}\,\tilde{\varepsilon}_{ij}^{f}\,dV_f \qquad \Phi_m = \frac{1}{2}\int_{V_m} \tilde{\boldsymbol{\sigma}}_{ij}^{m}\,\tilde{\varepsilon}_{ij}^{m}\,dV_m$$

$$\Phi_f = \frac{1}{2}\int_{V_f} C_{ijkl}\,\tilde{\varepsilon}_{ij}^{f}\,\tilde{\varepsilon}_{kl}^{f}\,dV_f \qquad \Phi_m = \frac{1}{2}\int_{V_m} C_{ijkl}\,\tilde{\varepsilon}_{ij}^{m}\,\tilde{\varepsilon}_{kl}^{m}\,dV_m$$

Assuming transverse isotropy and expanding in *i,j,k* and, *l* yields

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Expression for Interaction energy

$$\Phi_f = \frac{1}{2}\left[\begin{array}{l} C_{11}^{f}\left\langle\left(\tilde{\varepsilon}_{11}^{f}\right)^2\right\rangle + C_{22}^{f}\left\langle\left(\tilde{\varepsilon}_{22}^{f}\right)^2\right\rangle + C_{33}^{f}\left\langle\left(\tilde{\varepsilon}_{33}^{f}\right)^2\right\rangle + 2C_{12}^{f}\left\langle\tilde{\varepsilon}_{11}^{f}\cdot\tilde{\varepsilon}_{22}^{f}\right\rangle + 2C_{13}^{f}\left\langle\tilde{\varepsilon}_{11}^{f}\cdot\tilde{\varepsilon}_{33}^{f}\right\rangle + \\ 2C_{23}^{f}\left\langle\tilde{\varepsilon}_{22}^{f}\cdot\tilde{\varepsilon}_{33}^{f}\right\rangle + C_{12}^{f}\left\langle\left(\tilde{\gamma}_{12}^{f}\right)^2\right\rangle + C_{13}^{f}\left\langle\left(\tilde{\gamma}_{13}^{f}\right)^2\right\rangle + C_{23}^{f}\left\langle\left(\tilde{\gamma}_{23}^{f}\right)^2\right\rangle \end{array}\right]$$

$$\Phi_m = \frac{1}{2}\left[\begin{array}{l} C_{11}^{m}\left\langle\left(\tilde{\varepsilon}_{11}^{m}\right)^2\right\rangle + C_{22}^{m}\left\langle\left(\tilde{\varepsilon}_{22}^{m}\right)^2\right\rangle + C_{33}^{f}\left\langle\left(\tilde{\varepsilon}_{33}^{m}\right)^2\right\rangle + 2C_{12}^{m}\left\langle\tilde{\varepsilon}_{11}^{m}\cdot\tilde{\varepsilon}_{22}^{m}\right\rangle + 2C_{13}^{m}\left\langle\tilde{\varepsilon}_{11}^{m}\cdot\tilde{\varepsilon}_{33}^{m}\right\rangle + \\ 2C_{23}^{m}\left\langle\tilde{\varepsilon}_{22}^{m}\cdot\tilde{\varepsilon}_{33}^{m}\right\rangle + C_{12}^{m}\left\langle\left(\tilde{\gamma}_{12}^{m}\right)^2\right\rangle + C_{13}^{m}\left\langle\left(\tilde{\gamma}_{13}^{m}\right)^2\right\rangle + C_{23}^{m}\left\langle\left(\tilde{\gamma}_{23}^{m}\right)^2\right\rangle \end{array}\right]$$

$$\Delta U = \Phi_f V_f + \Phi_m V_m$$

How does it depend on fiber volume fraction , properties of the materials or applied load state?

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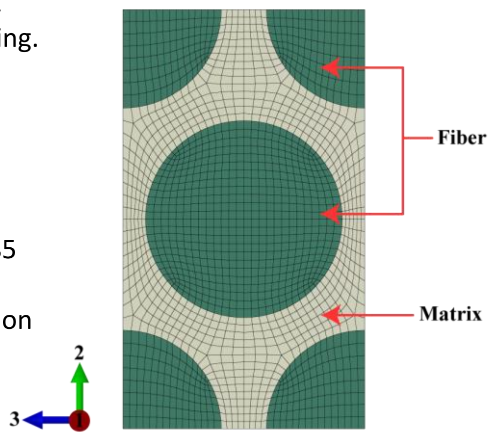
FEA model

- Representative Volume Element (RVE) with hexagonal fiber packing.
- Fiber material - Carbon

Three parametric studies:

1. Fiber VF varied from 0.05 to 0.85
2. Matrix modulus varied as function of fiber modulus
3. Five types of biaxial loads



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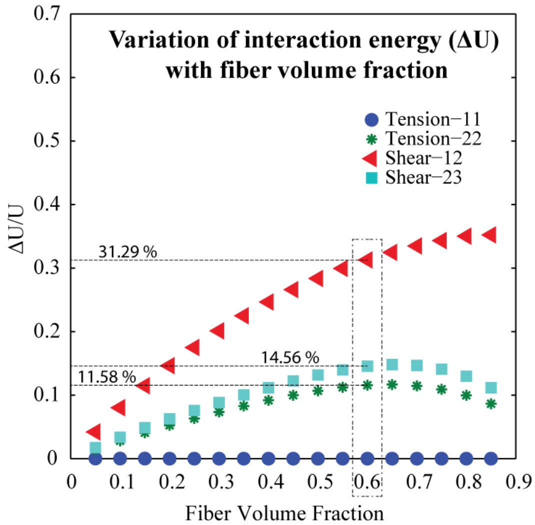
Effect of fiber volume fraction on interaction energy

- Matrix modulus 1% (2.35 GPa)
- Strongly dependent on the loading
- Maximum for shear-12 & negligible for tension-11
- For VF 0.6 ΔU is about 30% for shear-12.

Why does ΔU vary with fiber VF and load case ?



A closer look at the expression for interaction energy

- Material inhomogeneity increases and decreases with fiber VF

$$\Phi = \frac{1}{2}\left[\begin{array}{l} C_{11}\left\langle(\tilde{\varepsilon}_{11})^2\right\rangle + C_{22}\left\langle(\tilde{\varepsilon}_{22})^2\right\rangle + C_{33}\left\langle(\tilde{\varepsilon}_{33})^2\right\rangle + 2C_{12}\left\langle\tilde{\varepsilon}_{11}\cdot\tilde{\varepsilon}_{22}\right\rangle + 2C_{13}\left\langle\tilde{\varepsilon}_{11}\cdot\tilde{\varepsilon}_{33}\right\rangle + \\ 2C_{23}\left\langle\tilde{\varepsilon}_{22}\cdot\tilde{\varepsilon}_{33}\right\rangle + C_{12}\left\langle(\tilde{\gamma}_{12})^2\right\rangle + C_{13}\left\langle(\tilde{\gamma}_{13})^2\right\rangle + C_{23}\left\langle(\tilde{\gamma}_{23})^2\right\rangle \end{array}\right]$$

- $\Phi = f(\tilde{\varepsilon}) = f(\tilde{\sigma})$ and $\Delta U = \Phi_f V_f + \Phi_m V_m$
- So $\Delta U = f(\tilde{\varepsilon}) = f(\tilde{\sigma})$

Q1 : Negligible $\tilde{\varepsilon}/\tilde{\sigma}$ in Tension-11 Q2 : Maximum $\tilde{\varepsilon}/\tilde{\sigma}$ in Shear-12

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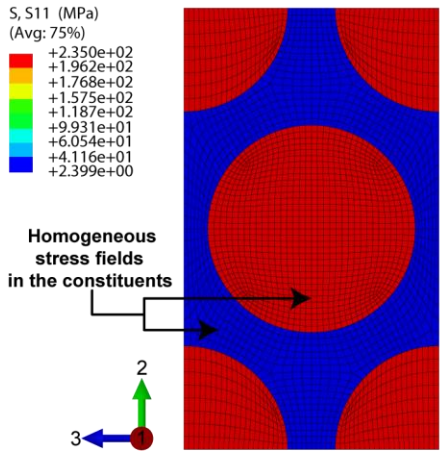
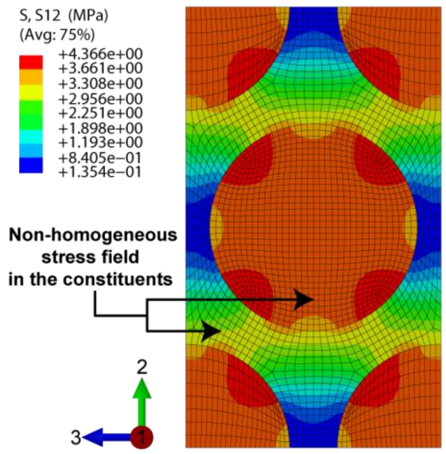
Distribution of stress with load case for fiber VF 0.6





ΔU in Tension-11 is negligible

ΔU in Shear-12 is about 30%

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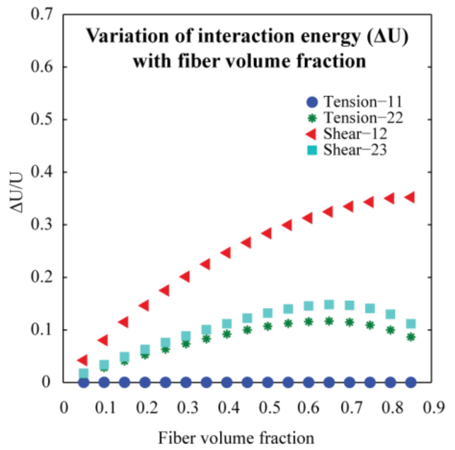
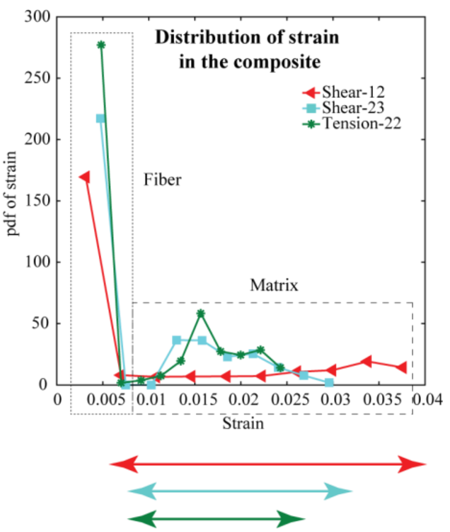
Interaction energy in tension-22 and shear-23





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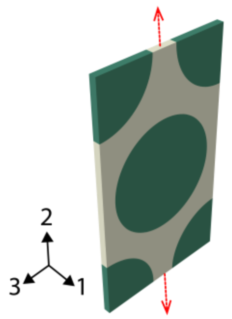
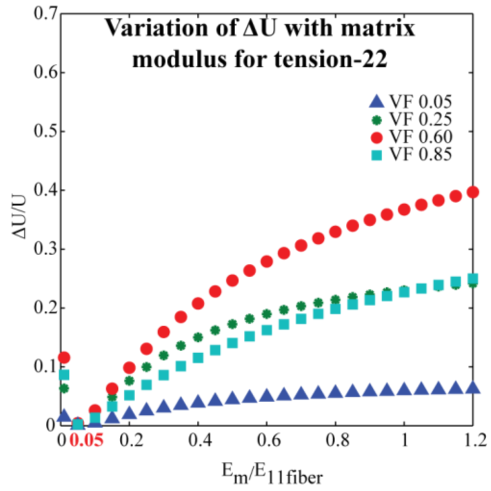
Effect of material properties on interaction energy



$E_{22_{matrix}} = 5\% \, E_{11_{fiber}}$
$E_{22_{matrix}} = 11.75 \; GPa$
$E_{22_{fiber}} = 14.00 \; GPa$

Interaction Energy is minimum



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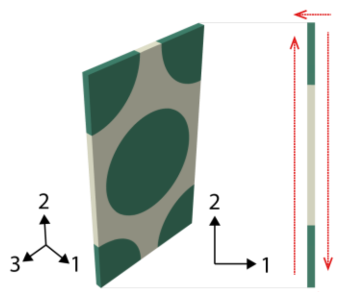
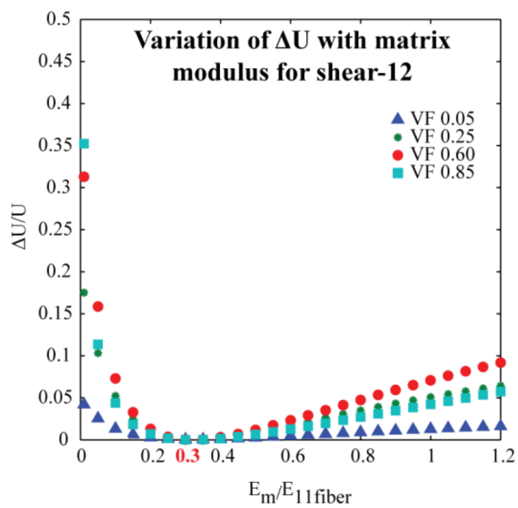
Effect of material properties on interaction energy



$E_{22_{matrix}} = 30\% \, E_{11_{fiber}} = 11.75 GPa$

$G_{12_{matrix}} = 26.31 \; GPa$
$G_{12_{fiber}} = 28.00 \; GPa$

Interaction Energy is minimum



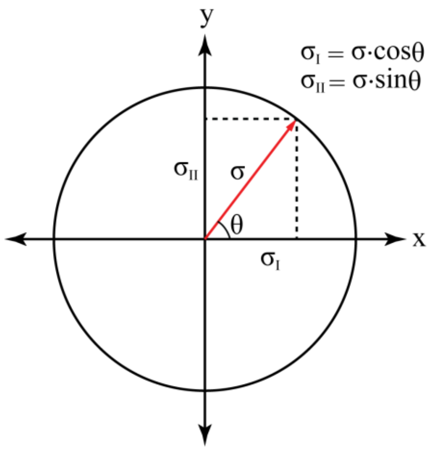
Effect of biaxial loading on interaction energy

- Five types of biaxial loads were considered

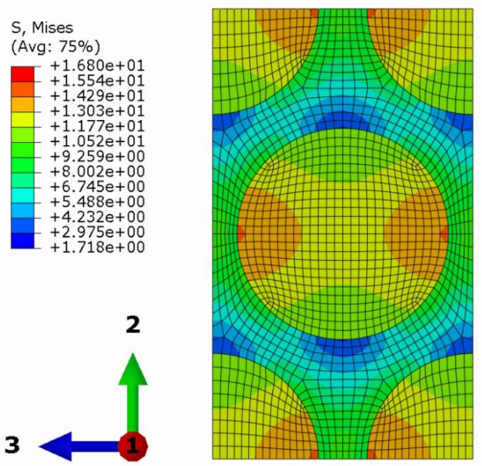
- $\sigma_{22} - \sigma_{33}$
- $\sigma_{12} - \sigma_{22}$
- $\sigma_{12} - \sigma_{23}$
- $\sigma_{12} - \sigma_{13}$
- $\sigma_{23} - \sigma_{22}$

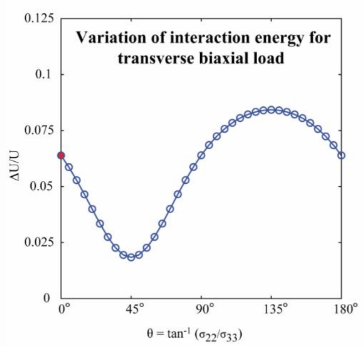
- Biaxial load represented by radius of circle

- θ is varied from 0 ° to 180°



Effect of transverse biaxial loading on interaction energy





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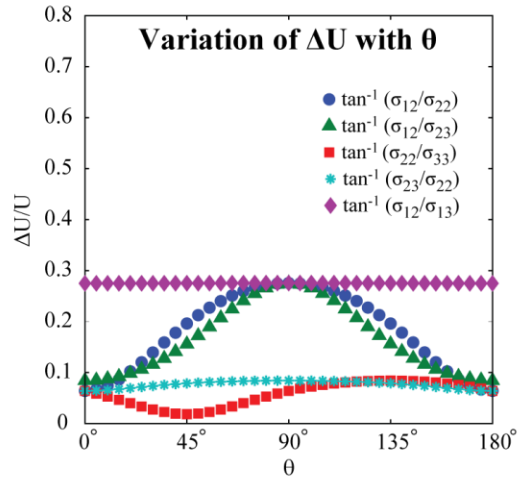
Effect of biaxial loading on interaction energy

- $E_m = 1.702\% \, E_f = 4.0 \, GPa$
- For $\sigma_{12} - \sigma_{13}$ interaction energy is constant
- For $\sigma_{22} - \sigma_{33}$ interaction energy is minimum at 45° and peaks at 135°
- For remaining three cases interaction energy is maximum at an angle of 90°



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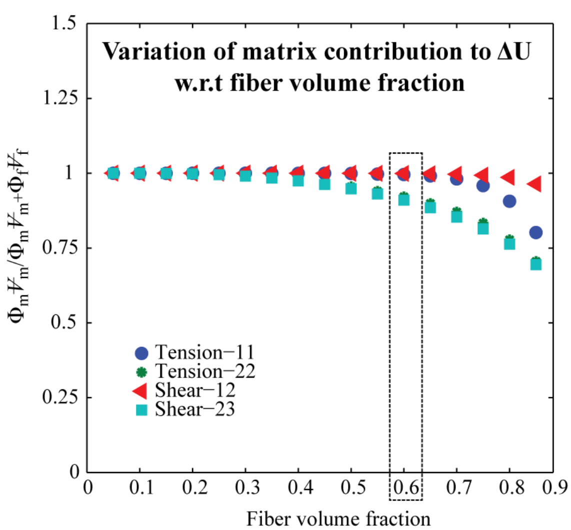
Major contributor to interaction energy



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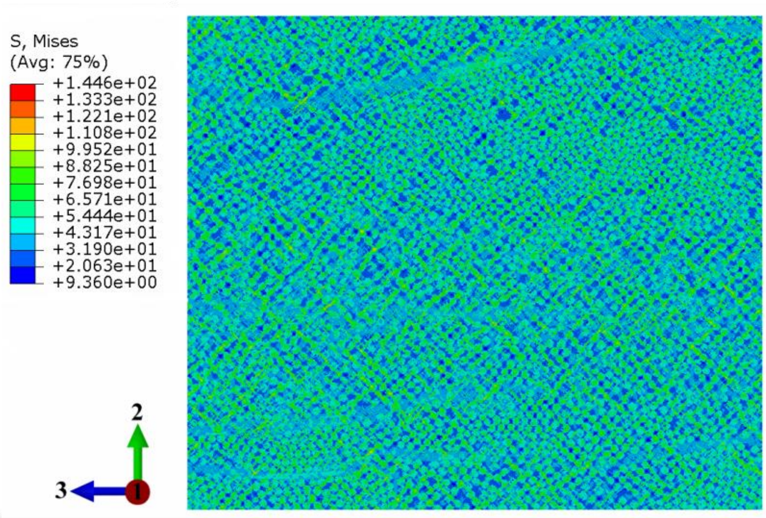
Stress plot of a real microstructure for load case shear-23



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Interaction energy of a real microstructure

Load case	Missing energy
Tension-11	4.043e-5
Tension -22	0.144
Tension - 33	0.154
Shear-12	**0.365**
Shear-13	**0.370**
Shear-23	0.168

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Conclusions

- Interaction energy is in the range of 30-40% for shear loading.
- All this interaction energy is due to the matrix
- Need to augment only the matrix failure theory with this energy

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Thank You.

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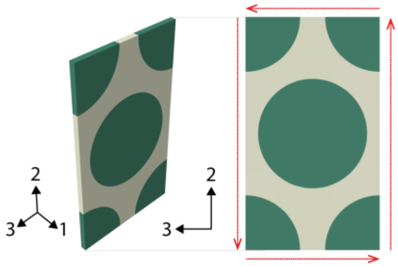
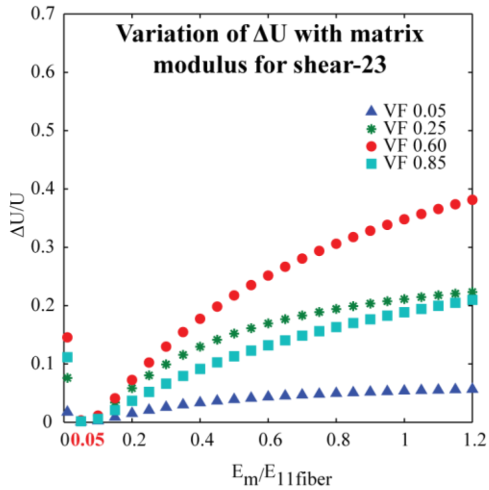
Effect of material properties on interaction energy





$E_{22_{matrix}} = 5\%\ E_{11_{fiber}} = 11.75GPa$

$G_{23_{matrix}} = 4.38\ GPa$

$G_{23_{fiber}} = 5.60\ GPa$

Interaction Energy is minimum

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Material properties

		Volume fraction variation	Matrix modulus variation	Biaxial loading
Material	Fiber	Matrix	Matrix	Matrix
Material type	Transversely isotropic	Isotropic	Isotropic	Isotropic
$E_{11}(GPa)$	235.0	$0.01E_{11}$ (2.35)	$0.01E_{11}$ *to* $1.2E_{11}$	4.0
$E_{22}\ (GPa)$	14.0	$0.01E_{11}$ (2.35)	$0.01E_{11}$ *to* $1.2E_{11}$	4.0
$G_{12}(GPa)$	28.0	0.8769	Varies with matrix modulus	1.493
ν_{12}	0.2	0.34	0.34	0.34
ν_{23}	0.25	0.34	0.34	0.34

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References

- Sun, C. T., and Vaidya, R. S. "Prediction of composite properties from a representative volume element," *Composites Science and Technology* Vol. 56, No. 2, 1996, pp. 171-179. doi: 10.1016/0266-3538(95)00141-7

Material properties of the real microstructure

	Fiber	Matrix
Material type	Transversely isotropic	Isotropic
$E_{11}(GPa)$	210.9	2.723 (1.29% E_{11})
$E_{22}\,(GPa)$	16.95	2.723 (1.29% E_{11})
$G_{12}(GPa)$	18.09	0.8769
ν_{12}	0.247	0.323
ν_{23}	0.197	0.323

Matrix failure theory

$$\boldsymbol{B_t\{I_t\}^2 + B_{s1}I_{s1} + B_{s2}I_{s2} = 1}$$

Where

$$I_t = \frac{\sigma_{22_m} + \sigma_{33_m} + \sqrt{\left(\sigma_{22_m} + \sigma_{33_m}\right)^2 - 4\left(\sigma_{22_m}\sigma_{33_m} + {\sigma_{23_m}}^2\right)}}{2}$$

$$I_{s1} = {\sigma_{12_m}}^2 + {\sigma_{13_m}}^2$$

$$I_{s2} = \frac{1}{4}\left(\sigma_{22_m} - \sigma_{33_m}\right)^2 + {\sigma_{23_m}}^2$$

- The {} denote Macaulay brackets.
- The values of B_i are determined from three composite static failure tests: transverse tension, transverse compression, and in-plane shear.

1. **A COMPUTATIONALLY EFFICIENT METHOD FOR MULTISCALE MODELING OF COMPOSITE MATERIALS: EXTENDING MULTICONTINUUM THEORY TO COMPLEX 3D COMPOSITES,** Ray S. Fertig, III, *Firehole Technologies.*